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An Interpolation-Free Cell-Centered Finite Volume Scheme for 3D Anisotropic Convection-Diffusion Equations on Arbitrary Polyhedral Meshes

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Abstract. Most existing cell-centered finite volume schemes need to introduce auxiliary unknowns in order to maintain the second-order accuracy when the mesh is distorted or the problem is discontinuous, so interpolation algorithms of auxiliary unknowns are required. Interpolation algorithms are not only difficult to construct, but also bring extra computation. In this paper, an interpolation-free cell-centered finite volume scheme is proposed for the heterogeneous and anisotropic convection-diffusion problems on arbitrary polyhedral meshes. We propose a new interpolation-free discretization method for diffusion term, and two new second-order upwind algorithms for convection term. Most interestingly, the scheme can be adapted to any mesh topology and can handle any discontinuity strictly. Numerical experiments show that this new scheme is robust, possesses a small stencil, and has approximately second-order accuracy for both diffusion-dominated and convection-dominated problems.

AMS subject classifications: 65M08, 65M22

Key words: Interpolation-free, finite volume scheme, convection-diffusion, polyhedral mesh.

1 Introduction

The heterogeneous and anisotropic convection-diffusion equation is an important part of many complex problems, such as radiation hydrodynamics, semiconductor device modeling, petroleum engineering and groundwater flow. Therefore, the design of numerical schemes for convection-diffusion equation is the cornerstone in studying these complex problems. With the development of computer, physical modeling is becoming more and

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more accurate, and numerical simulation is rapidly developing from two-dimensional (2D) to three-dimensional (3D). It is very important and meaningful to construct a robust and accurate scheme for the heterogeneous and anisotropic convection-diffusion equations on arbitrary polyhedral meshes.

Conservation is a very important physical property of the convection-diffusion equation. We naturally hope that the constructed numerical scheme also has conservation properties, so that the numerical solution is consistent with physics. Generally, the mesh is highly distorted in many practical applications. The highly anisotropic diffusion and strong discontinuity also bring a lot of trouble to the construction of numerical schemes. Convection-dominated is another difficulty, which may lead to numerical oscillations or even solver crashes. A desirable scheme should be applicable to the diffusion-dominated case, to the convection-dominated case, and to all cases between. The above facts are the main difficulties in constructing the numerical schemes for the heterogeneous and anisotropic convection-diffusion equations.

In recent years, numerous work has been devoted in developing efficient numerical methods for convection-diffusion equations, such as the finite difference method [1], the finite element method [2–6], the finite volume element method [7, 8], the finite volume method [9, 10], etc. The finite volume method is widely used because of its simplicity and local conservation. For the calculation of diffusion term, some linear finite volume schemes [11–17] and nonlinear finite volume schemes [18–25] have been developed for heterogeneous and anisotropic diffusion equations in recent years. For a more systematic overview, we refer the reader to, e.g., [26, 27]. Here, we focus on the cell-centered finite volume schemes for the convection-diffusion equations. Although there are some schemes, such as the hybrid mimetic mixed method and the hybrid high order schemes, which can easily capture anisotropy and heterogeneity, and handle advection-dominated regimes [26], it seems not so easy for cell-centered finite volume schemes. In 1999, Y. Coudiere et al. proposed a finite volume scheme of diamond type to solve the convection-diffusion equation in [28]. The convection term is approximated by a first-order upwind algorithm, and the auxiliary unknowns defined at vertices are eliminated by the least square method. Then, the convergence rate of the scheme on the locally refined mesh was analyzed in [29]. Further developments of the diamond scheme can be found in [30–32]. In [33], a monotone finite volume scheme for 2D convection-diffusion equation was developed, where the vertex auxiliary unknowns are eliminated by the interpolation algorithm proposed by [15]. If the interpolation coefficient is negative, then the interpolation algorithm will be replaced by a low-order positive-preserving one as in [19]. In [34], an extremum-preserving finite volume scheme was proposed, where the harmonic averaging point was used as the auxiliary unknown. Since the harmonic averaging points may be located outside the corresponding faces, it is possible for this scheme to lose accuracy in some extreme cases. In addition, high order finite volume schemes were studied in [35–37], but only scalar diffusion coefficients were considered. In [38], a non-overlapping domain decomposition algorithm for solving convection-diffusion equations with finite volume scheme was studied. Recently, the long-time behavior of