Journal of Computational Mathematics
Vol.42, No.1, 2024, 248–270.
http://www.global-sci.org/jcm
doi:10.4208/jcm.2206-m2021-0354

STRONG CONVERGENCE OF JUMP-ADAPTED IMPLICIT MILSTEIN METHOD FOR A CLASS OF NONLINEAR JUMP-DIFFUSION PROBLEMS*

Xu Yang
School of Mathematics, China University of Mining and Technology, Xuzhou 221116, China
Email: xuyang96@cumt.edu.cn
Weidong Zhao[1)]
School of Mathematics, Shandong University, Jinan 250100, China
Email: wdzhao@sdu.edu.cn

Abstract

In this paper, we study the strong convergence of a jump-adapted implicit Milstein method for a class of jump-diffusion stochastic differential equations with non-globally Lipschitz drift coefficients. Compared with the regular methods, the jump-adapted methods can significantly reduce the complexity of higher order methods, which makes them easily implementable for scenario simulation. However, due to the fact that jump-adapted time discretization is path dependent and the stepsize is not uniform, this makes the numerical analysis of jump-adapted methods much more involved, especially in the non-globally Lipschitz setting. We provide a rigorous strong convergence analysis of the considered jump-adapted implicit Milstein method by developing some novel analysis techniques and optimal rate with order one is also successfully recovered. Numerical experiments are carried out to verify the theoretical findings.

Mathematics subject classification: 60H35, 65C30, 65L20.
Key words: Jump-diffusion, Jump-adapted implicit Milstein method, Poisson jumps, Strong convergence rate, Non-Lipschitz coefficients.

1. Introduction

In this paper, we consider numerical solution of jump-diffusion Itô stochastic differential equations of the form

$$\begin{cases} \mathrm{d}X_t = f(X_{t-})\mathrm{d}t + g(X_{t-})\mathrm{d}W_t + h(X_{t-})\mathrm{d}N_t, \; t \in \big(0,T\big], \\ X_0 = x_0, \end{cases} \tag{1.1}$$

where $T > 0$ is a fixed constant, X_{t-} denotes $\lim_{s\uparrow t} X_s$, $f\colon \mathbb{R}^m \to \mathbb{R}^m$ is the drift coefficient, $g\colon \mathbb{R}^m \to \mathbb{R}^{m\times d}$ is the diffusion coefficient which is frequently written as $g = (g_{i,j})_{m\times d} = (g_1, g_2, \cdots, g_d)$ for $g_{i,j}\colon \mathbb{R}^m \to \mathbb{R}$ and $g_j\colon \mathbb{R}^m \to \mathbb{R}^m$, $j \in \{1,\ldots,d\}$, $h\colon \mathbb{R}^m \to \mathbb{R}^m$ is the jump coefficient, with $m, d \in \mathbb{N}^+$. Here W_t is a d-dimensional Wiener process and N_t is a scalar Poisson process with intensity $\lambda > 0$, both defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with a normal filtration $\mathbb{F}\colon = \{\mathcal{F}_t\}_{t\in[0,T]}$, and they are independent with each other. Specific conditions on the coefficients f, g, h and the initial value x_0 will be given in next section.

* Received December 15, 2021 / Revised version received January 12, 2022 / Accepted June 14, 2022 / Published online February 22, 2023 /
1) Corresponding author

Jump-diffusion stochastic differential equations (JSDEs) have found many applications in biology, physics, chemistry, engineering, finance, and insurance. Especially in the area of finance and insurance, jump-diffusion models are often used to describe the dynamic evolution of various state variables, such as stock indices, asset prices, interest rates, credit ratings, exchange rates or commodity prices. In these models, the jump driving part is often used to characterize and capture the event-driven uncertainties and the unexpected abrupt changes, such as corporate defaults, operational failures or insured accidents. For more details on the application of jump-diffusion stochastic differential equations in the financial field, we refer the reader to the books [9, 32].

Since most of JSDEs cannot be solved explicitly, it is necessary and important to develop discrete time approximations to study the behavior of jump-diffusion models and solve practical application problems. In view of the above reason, the study of numerical approximation of JSDEs has been attracting lots of attention. Generally, the discrete time approximations of JSDEs are divided into regular and jump-adapted schemes. Regular schemes employ time discretizations that do not include the jump times of the Poisson process, while jump-adapted schemes are based on jump-adapted time discretizations which include these jump times.

As far as the regular schemes are concerned, up to now, a lot of research results have been achieved, see [1, 3–5, 11, 14, 17–19, 32–34, 37] and the references therein. However, in the vast majority of the above mentioned research articles, a global Lipschitz assumption on the coefficients of JSDEs is often used. Note that the assumption of global Lipschitz continuity is very restrictive and many practical problems fail to satisfy such condition. In fact, the coefficients of numerous JSDEs used to describe financial models are either super-linear growth or sub-linear growth, which are obviously not globally Lipschitz continuous. Therefore, the theoretical analysis basis of many existing numerical methods has changed. This makes the scope of application of those numerical methods whose theoretical framework are based on global Lipschitz condition, greatly reduced, and those numerical methods even no longer be applicable. Therefore, in recent years, many scholars have begun to turn to dealing with numerical approximation of JSDEs under some weaker conditions, such as local Lipschitz condition, one-sided Lipschitz condition, and great progress has been made, see [6–8, 10, 12, 13, 15, 20–23, 25, 27, 35, 36, 38, 39, 41].

To avoid the generation of multiple stochastic integrals with respect to the Poisson process, jump-adapted approximations were first introduced in [31]. As the jump-adapted time discretizations include all jump times generated by the Poisson process, the form of the resulting schemes is much simpler than that of the regular schemes, significantly reducing the complexity of higher order schemes. Hence the jump-adapted time discretization makes these corresponding schemes easily implementable for scenario simulation. Up to now, various jump-adapted numerical methods have been formulated and analyzed in [2, 24, 26, 28, 29, 32] and references therein. Nevertheless, like the regular case, most of convergence results for jump-adapted approximations are always analyzed in the globally Lipschitz setting, and there exist only a very limited number of works devoted to the numerical study of jump-adapted schemes under the non-global Lipschitz condition. A transformed jump-adapted backward Euler method for a class of jump-diffusion financial models whose coefficients do not satisfy the global Lipschitz condition was studied in [40].

The main objective of this paper is to study the strong convergence of a jump-adapted implicit Milstein method for a class of general jump-diffusion models, for which we require that the drift coefficient is one-sided Lipschitz continuous, and the diffusion coefficient and the jump coefficient are globally Lipschitz continuous (see Assumption 2.1 in the next section). Due to