INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING, *SERIES B*
Volume 5, Number 1-2, Pages 66–78

STABILITY AND NUMERICAL DISPERSION ANALYSIS OF FINITE-DIFFERENCE METHOD FOR THE DIFFUSIVE-VISCOUS WAVE EQUATION

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Abstract. The diffusive-viscous wave equation plays an important role in seismic exploration and it can be used to explain the frequency-dependent reflections observed both in laboratory and field data. The numerical solution to this type of wave equation is needed in practical applications because it is difficult to obtain the analytical solution in complex media. Finite-difference method (FDM) is the most common used in numerical modeling, yet the numerical dispersion relation and stability condition remain to be solved for the diffusive-viscous wave equation in FDM. In this paper, we perform an analysis for the numerical dispersion and Von Neumann stability criteria of the diffusive-viscous wave equation for second order FD scheme. New results are compared with the results of acoustic case. Analysis reveals that the numerical dispersion is inversely proportional to the number of grid points per wavelength for both cases of diffusive-viscous waves and acoustic waves, but the numerical dispersion of the diffusive-viscous waves is smaller than that of acoustic waves with the same time and spatial steps due to its more restrictive stability condition, and it requires a smaller time step for the diffusive-viscous wave equation than acoustic case.

Key words. Stability, dispersion analysis, finite-difference method, diffusive-viscous wave equation, acoustic waves

1. Introduction

The diffusive-viscous wave equation was proposed recently in the field of oil and gas exploration. The low-frequency seismic anomalies related to hydrocarbon reservoirs have lately attracted wide attention [25, 7, 17, 8]. Even though the relationship between the frequency-dependent reflections and fluid saturation in a reservoir can be quite complex, but there is a general connection between the character of porous medium saturation and seismic response. Goloshubin and Bakulin observed phase shifts and energy redistribution between different frequencies when comparing cases of water-saturated and gas-saturated rocks [14, 12]. Korneev et al. observed that reflections from a fluid-saturated layer have increased amplitude and delayed traveltime at low frequencies when compared with reflections from a dry layer in both laboratory and field data [17]. Those observed results cannot be explained using Biot theory [12, 3, 4, 5, 21, 10, 2], nor by the reflection properties of an elastic layer [17], or the squirt flow and patchy saturation models [20]. Korneev et.al. proposed a diffusive-viscous model to explain the frequency-dependent phenomena in fluid-saturated porous reservoirs [17]. Therefore, the diffusive-viscous theory is important in seismic exploration, for example, it can be used for detecting and monitoring hydrocarbon reservoirs [15], and it is also essential to simulate the propagation of the diffusive-viscous waves in practical applications.

Seismic numerical modeling is a valuable tool for seismic interpretation and an essential part of seismic inversion algorithms. Another important application of seismic modeling is the evaluation and design of seismic surveys [6]. There are

Received by the editors February 2, 2014 and, in revised form, March 20, 2014.
2000 *Mathematics Subject Classification.* 35R35, 49J40, 60G40.
This research was supported by National Natural Science Foundation of China (40730424) and National Science & Technology Major Project (2011ZX05023-005).

many approaches to seismic modeling. The finite-difference method(FDM) is the most straightforward numerical approach in seismic modeling, and it is also becoming increasingly more important in the seismic industry and structural modeling due to its relative accuracy and computational efficiency [22]. Some of the most common FDMs used in seismic modeling are explicit, and thus conditionally stable. Generally in seismology, explicit methods are preferred over implicit ones because they need less computation at each time step and have the same order of accuracy. This has been noted for FDM [6, 11]. However, the size of the time step is bounded by a stability criterion which is an important factor affecting the accuracy of the results. Additionally, a numerical dispersion (grid dispersion) related to grid spacing has a detrimental effect on accuracy of FD scheme. It occurs because the actual velocity of high-frequency waves in the grid is different from the true velocity and it can occur even when the physical problem is not dispersive [9]. The error introduced by numerical dispersion is dependent on the grid spacing and the size of the time step. There are many studies in literature regarding the numerical dispersion and stability analysis for acoustic wave propagation [1, 19]. However, the numerical dispersion analysis and stability condition is rarely seen despite its significance in seismic exploration for the diffusive-viscous wave propagation.

Our aims in this paper are to estimate the Von Neumann stability criteria and derive the numerical dispersion relation for the finite-difference method for the diffusive-viscous wave equation proposed by Korneev [17]. We will show that there are some differences of stability condition and dispersion relation between the diffusive-viscous wave equation and acoustic wave equation, and the dispersion of diffusive-viscous waves is smaller than that of acoustic waves with the same time and spatial steps because of its more restrictive stability condition, and it requires a smaller time step for the diffusive-viscous wave equation than acoustic case.

2. The diffusive-viscous theory

In this section, we will first introduce the diffusive-viscous wave equation, then give the propagating wavenumber and attenuation coefficient of the diffusive-viscous waves prepared for the following section.

2.1. The diffusive-viscous wave equation. The diffusive-viscous theory is proposed by Korneev [17, 13], which is used to explain the relationship between the frequency dependence of reflections and the fluid saturation in a reservoir. The diffusive-viscous wave equation in a 1-D medium is mathematically described as follows:

$$\frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t} - \eta \frac{\partial^3 u}{\partial x^2 \partial t} - \upsilon^2 \frac{\partial^2 u}{\partial x^2} = 0 \tag{1}$$

for $(x, t) \in (-\infty, \infty) \times [0, \infty)$, where u is the wave field; $\gamma \geq 0$, $\eta \geq 0$ are diffusive and viscous attenuation parameters, respectively, which are the functions of the porosity and the permeability of reservoir rocks and the viscosity and the density of the fluid; υ is the wave propagation velocity in a non-dispersive medium. The second term in (1) characterizes a diffusional dispersive force, whereas the third term describes the viscosity. t is the time and x is the space variables. Equation (1) is extended to two dimensional case (2-D) as [15]

$$\frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t} - \eta (\frac{\partial^3 u}{\partial x^2 \partial t} + \frac{\partial^3 u}{\partial z^2 \partial t}) - \upsilon^2 (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}) = 0 \tag{2}$$

The definitions of the variables are the same as (1), and $(x, z) \in (-\infty, \infty) \times (-\infty, \infty)$ are the Cartesian coordinates.