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The Effect of Clone Template Parameters on the Spreading Speeds in CNNs*

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Abstract The aim of this paper is to investigate the effect of clone template parameters on the spreading speeds in cellular neural networks(CNNs). According to the property analysis of spreading speeds of monotone semiflows developed by Yu and Zhang [*European Journal of Applied Mathematics*, **31** (2020), 369-384], we investigate the sign of spreading speeds, continuity and limit cases with no propagation phenomena for CNNs with general output functions where each cell interacts with its 2-neighborhood cell.

Keywords Sign of spreading speeds, continuity, CNNs

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1. Introduction

The aim of this paper is to investigate the effect of parameters on the spreading speeds in CNNs. Cellular neural network is a large-scale nonlinear simulation processor that is locally connected and capable of real-time signal processing, proposed by Chua and Yang [2] in 1988. Each of its basic circuit units is a cell neuron, which is regularly connected by the same cell neurons in space. These cell neurons only contact and interact with neighboring cell neurons, and each neuron has internal states related to input, output and dynamic rules. It has the characteristics of continuous real-time, high speed parallel computing and Very Large Scale Integration (VLSI). Over the past 20 years, the research results of CNNs have been widely applied in many fields, such as biomedicine, image processing, automatic control and pattern recognition. The circuit model of one-dimensional standard CNN without input is

$$\frac{dx_n(t)}{dt} = -x_n(t) + z + \sum_{k\in N_r(n)} A(n,k)f(x_k),\ n \in \mathbb{Z}. \tag{1.1}$$

In the above expression, the node voltage x_n at point n is called the state of the cell neuron at point n. The quantity z is called the threshold term or the offset term and is associated with an independent voltage source in the circuit. The output

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function f(a nonlinear function) is given by

$$f(x) = \frac{1}{2}(|x+1| - |x-1|). \tag{1.2}$$

For a positive integer r, the r-neighborhood $N_r(n)$ of a cell at n is defined as

$$N_r(n) = \{k \in \mathbb{Z} : \ |k-n| \le r\}.$$

For each n and $k \in N_r(n)$, $A(n,k)$ constitutes the so-called clone template, which measures the coupling weights of the cells at n from the cells at k and specifies the interactions between each cell and all of its neighbors in terms of the state and output variables. When the template is the space-invariant, each cell is described by the simple identical cloning template, i.e. $A(n, n+k) \equiv A(0,k) := a_k$ $(k \in N_r(0))$ or $A(n,k) = A(n-k)$ $(k \in N_r(n))$. If $r = 1$, letting $a_k := A(0,k)$ $(k \in N_1(0))$, then these numbers can be arranged in a 1×3 matrix form $A := [a_{-1}, a_0, a_1]$ and (1.1) can be written by

$$\frac{dx_n(t)}{dt} = -x_n(t) + z + a_{-1} f(x_{n-1}) + a_0 f(x_n) + a_1 f(x_{n+1}), \ n \in \mathbb{Z}. \tag{1.3}$$

In CNNs, several experimental studies have revealed the propagation of activity in sections of excitable nerve tissue. The basic mechanism for the propagation of these waves (i.e., traveling waves) is thought to originate at synapses, rather than disperse like the propagation of action potentials. Since then,the study of the CNN equation has been extended to more general equations, and the propagation of these waves has also been widely investigated(see, e.g. [4–7, 10, 12, 14–16, 19, 20]). Wu and Hsu [14, 15] considered the existence of the entire solutions for CNNs. Yu and Zhao [17] investigated the propagation phenomena of monotone and non-monotone CNNs with asymmetric templates and distribution delays.

Recently, Yu and Zhang [18] have studied the properties of spreading speeds and have obtained a general method for analyzing the sign, continuity, and limit cases with no propagation phenomena for monotone semiflows. These results are applied to CNNs (1.3) with $z = 0$ where each cell interacts with its 1-neighborhood cell and the output function f satisfies (1.2). Moreover, three different propagation phenomena are determined according to the clone template parameters from 1-neighborhood cells. More recently, Bai and Yang [1] have studied the influence of parameters on the spreading speeds of CNNs ($r = 1$) with time delay. Therefore, motivated by the work of Yu and Zhang [18], we will further consider the influence of interaction parameters on the spreading speeds for the CNNs with general output functions where each cell interacts with its 2-neighborhood cells. More precisely, we investigate the following CNNs with a general output function

$$\begin{aligned} \frac{dx_i}{dt} =& - x_i(t) + \alpha_2 f(x_{i-2}(t)) + \alpha_1 f(x_{i-1}(t)) + a f(x_i(t)) \\ & + \beta_1 f(x_{i+1}(t)) + \beta_2 f(x_{i+2}(t)), \end{aligned} \tag{1.4}$$

where $\alpha_2, \alpha_1, a, \beta_1, \beta_2$ are nonnegative and the output function f satisfies the following assumptions

(F1) There is $K > 0$ such that $(\alpha_2 + \alpha_1 + a + \beta_1 + \beta_2) f(K) = K$ and $f(0) = 0$; $f \in C([0, K], [0, \frac{K}{\alpha_2+\alpha_1+a+\beta_1+\beta_2}])$, $(\alpha_2 + \alpha_1 + a + \beta_1 + \beta_2) f'(0) > 1$, $|f(u) - f(v)| \le f'(0)|u - v|$ for $u, v \in [0, K]$.