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Dynamics of a Discrete Two-Species Competitive Model with Michaelies-Menten Type Harvesting in the First Species*

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Abstract In this paper, we use a semidiscretization method to derive a discrete two-species competitive model with Michaelis-Menten type harvesting in the first species. First, the existence and local stability of fixed points of the system are investigated by employing a key lemma. Subsequently, the transcritical bifurcation, period-doubling bifurcation and pitchfork bifurcation of the model are investigated by using the Center Manifold Theorem and bifurcation theory. Finally, numerical simulations are presented to illustrate corresponding theoretical results.

Keywords Competitive model with Michaelis-Menten type harvesting, semidiscretization method, transcritical bifurcation, period-doubling bifurcation, pitchfork bifurcation

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1. Introduction and preliminaries

In the past few decades, more and more investigators have begun to pay attention to investigating competitive systems [1, 2, 4–6, 9–12, 15, 19, 24–26, 29, 30, 32–34], and many excellent results concerned with the extinction and global attractivity of competitive systems have been obtained.

Murray [17] investigated the competitive system of traditional two-species Lotka-Volterra model

$$\begin{cases} \frac{dx_1}{dt} = x_1(b_1 - a_{11}x_1 - a_{12}x_2), \\ \frac{dx_2}{dt} = x_2(b_2 - a_{21}x_1 - a_{22}x_2), \end{cases} \tag{1.1}$$

where x_1 and x_2 denote the population density of the two species at time t respectively, and $b_i, a_{ij}, i, j = 1, 2$, are positive constants.

In addition, when human activity is the main cause which leads to the extinction of endangered species, the study of resource-management, including fisheries, forestry, and wildlife management, has great importance. It is sometimes necessary

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to harvest some populations, but harvesting should be regulated so that both the ecological sustainability and conservation of the species can be implemented in a long running. In order to further understand the scientific management of renewable resources and make the meaning of a model more realistic, many scholars are devoted to establishing suitable biological models. Among them, Chen [3] studied the following model

$$\begin{cases} \frac{dx}{dt} = r_1 x(1 - \frac{x}{k_1} - \alpha\frac{y}{k_1}) - \frac{qEx}{m_1E+m_2x}, \\ \frac{dy}{dt} = r_2 y(1 - \frac{y}{k_2}), \end{cases} \tag{1.2}$$

where x and y denote the population density of the first and second species at time t respectively, q denotes the fishing coefficient of the first species, E denotes the fishing effort, and $r_1, r_2, k_1, k_2, \alpha, m_1, m_2$ are all positive constants. The function $h(x) = \frac{qEx}{m_1E+m_2x}$ is called Michaelis-Menten type harvesting, which was proposed by Clark and Mangel [7]. In other pieces of literature, $h(x)$ may also take qEx, $\frac{qE}{m}$ or $\frac{qx}{m}$.

Later, in [31], based on model (1.2), Yu, Zhu and Li considered the following system:

$$\begin{cases} \frac{dx}{dt} = r_1 x(1 - \frac{x}{k_1}) - \alpha_1 xy - \frac{q_1Ex}{m_1E+h_1x}, \\ \frac{dy}{dt} = r_2 y(1 - \frac{y}{k_2}) - \alpha_2 xy, \end{cases} \tag{1.3}$$

where $r_1, r_2, k_1, k_2, \alpha_1, \alpha_2, q_1, m_1, h_1$ and E are all positive. For simplicity, the authors made the following nondimensional scheme:

$$\bar{t} = r_1 t, \bar{x} = \frac{1}{k_1}x, \bar{y} = \frac{1}{k_2}y.$$

Dropping the bars, system (1.3) becomes

$$\begin{cases} \frac{dx}{dt} = x(1 - x - a_1 y - \frac{b}{c+x}), \\ \frac{dy}{dt} = \rho y(1 - y - a_2 x), \end{cases} \tag{1.4}$$

where $a_1 = \frac{\alpha_1 k_2}{r_1}, b = \frac{q_1 E}{k_1 r_1 h_1}, c = \frac{m_1 E}{h_1 k_1}, \rho = \frac{r_2}{r_1}, a_2 = \frac{k_1 \alpha_2}{r_2}$.

Generally speaking, it is impossible to obtain an exact solution for a complex differential equation system. Therefore, one usually derives its approximate solution by using computer. Then, we should study its corresponding discrete model. For a given system, there are many discretization methods including Euler forward difference scheme, Euler backward difference scheme, semidiscretization methods and etc. In this article, we use the semidiscretization method, which has been applied in many studies ([8, 13, 14, 21]). For the related work, please also see [16, 18, 20, 27, 28].

The discrete version of system (1.4) has not been found to be investigated yet. Now, we use the semidiscretization method to derive its discrete model. For this, suppose that $[t]$ denotes the greatest integer not exceeding t. We consider the average change rate of system (1.4) at integer number points

$$\begin{cases} \frac{1}{x(t)}\frac{dx(t)}{dt} = 1 - x([t]) - a_1 y([t]) - \frac{b}{c+x([t])}, \\ \frac{1}{y(t)}\frac{dy(t)}{dt} = \rho(1 - y([t]) - a_2 x([t])). \end{cases} \tag{1.5}$$