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Stability and Bifurcation Analysis in a Nonlocal Diffusive Predator-prey Model with Hunting Cooperation*

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Abstract In this paper, we propose a diffusive predator-prey model with hunting cooperation and nonlocal competition. Under a rather general selection of the kernel function, we first study the stability of the positive equilibrium of the model. Then, we obtain the conditions which Hopf bifurcation and Turing bifurcation occur. Our results show that nonlocal competition plays an important role in determining the dynamics of the model.

Keywords Predator-prey system, Nonlocal competition, Stability, Hopf bifurcation, Turing bifurcation.

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1. Introduction

Recently, the predator-prey models with hunting cooperation have been widely studied by many researchers in the literature such as [1, 5, 9, 13, 16–21, 23, 24] for their importance in the real world. For instance, to better understand the impact of cooperative hunting upon the two trophic-level interactions, Alves and Hilker [1] proposed the following model with hunting cooperation in predators

$$\begin{cases} \frac{du}{dt} = ru(1 - \frac{u}{K}) - (\lambda + av)uv, \\ \frac{dv}{dt} = ev(\lambda + av)u - mv, \end{cases} \tag{1.1}$$

where $u(t)$ and $v(t)$ represent prey and predator densities at the time t respectively, r is the per capita intrinsic growth rate of prey, K is the carrying capacity of prey, e is the conversion efficiency and m is the per capita mortality rate of predators. λ is the attack rate per predator and prey, and a is a parameter describing predator cooperation in hunting. All parameters are positive. They investigated the existence and stability of the positive equilibrium, and showed that hunting cooperation is beneficial to the predator population by the increasing attack rate. Introducing Allee effect into model (1.1), Jang, Zhang and Larriva [9] investigated the existence and stability of the positive equilibrium, and presented the optimal control problem by numerical simulations. Then, not only the impact of hunting cooperation among

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predators but also predator-induced fear in prey population was considered by Pal et al. [13]. Sen, Ghorai and Banerjee [17] proposed a predator-prey model with Allee effect in prey growth rate and applied Holling type II functional response mechanism to describe the hunting cooperation. It is shown that the strong Allee effect in prey growth rate is able to strengthen the stability of the coexisting steady state.

More recently, the diffusion terms d_1 and d_2 have been introduced into model (1.1) and the corresponding diffusive model

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 \Delta u + u(r(1-\frac{u}{K}) - (\lambda + av)v), \\ \frac{\partial v}{\partial t} = d_2 \Delta v + v(e(\lambda + av)u - m) \end{cases} \tag{1.2}$$

has been studied by several scholars. Capone et al. [5] investigated the stability of the coexistence equilibria and obtained the conditions of Turing instability. Ryu and Ko [16] also obtained the asymptotic behaviour of positive steady state solutions when the cooperation effect of the predators is strong. The stability of positive constant steady state solution, Hopf bifurcation and Turing instability were studied by Wu and Zhao [23]. It is shown that the spatial model (1.2) can reserve the stability of the positive constant steady state solution, when the predation diffusion is not smaller than the prey diffusion. The complex patterns, such as spotted pattern, stripe pattern and mixed pattern, were obtained by Singh, Dubey and Mishra [19]. The results showed the effect of hunting cooperation in pattern dynamics of the diffusive model. Most recently, Song et al. [20] introduced the cross-diffusion into (1.2), and studied the stability and cross-diffusion driven Turing instability.

In addition, Singh and Banerjee [18] incorporated diffusion and Holling type II functional response in the predator-prey model with cooperative behavior in predators and also obtained complex patterns. The properties of the model were investigated by using extensive numerical simulations. Song et al. [21] considered a diffusive predator-prey model where the functional response follows the predator cooperation in hunting and the growth of the prey obeys the Allee effect. They investigated the diffusion-driven Turing instability, and derived the amplitude equation of Turing bifurcation by employing the weakly nonlinear analysis method. Wu and Song [24] introduced self-diffusion into the predator-prey model with hunting cooperation. Their research showed that Turing instability is induced by diffusion, and the conditions for Turing bifurcation to occur have been obtained.

For simplicity, we introduce the nondimensional parameters into model (1.2)

$$\sigma = \frac{r}{m} > 0, \quad \beta = \frac{e\lambda K}{m} > 0, \quad \alpha = \frac{am}{\lambda^2} \geq 0, \quad \bar{d_1} = \frac{1}{m} d_1, \quad \bar{d_2} = \frac{1}{m} d_2$$

and the other nondimensional variables

$$\bar{u} = \frac{e\lambda}{m} u, \quad \bar{v} = \frac{\lambda}{m} v, \quad \bar{t} = mt.$$

For the simplicity of notations, dropping the over-bars, then model (1.2) becomes

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 \Delta u + u(\sigma(1-\frac{u}{\beta}) - (1 + \alpha v)v), \\ \frac{\partial v}{\partial t} = d_2 \Delta v + v((1 + \alpha v)u - 1). \end{cases} \tag{1.3}$$