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Dynamics of a Stochastic SIR Epidemic Model with Logistic Growth*

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Abstract In this paper, a stochastic SIR epidemic model with saturated treatment function, non-monotone incidence rate and logistic growth is studied. First, we prove that the stochastic model has a unique global positive solution. Next, by constructing a suitable Lyapunov function, we can show that there exists an ergodic stationary distribution in the random SIR model. Then, we show that a sufficient condition can make the disease tend to extinction. Finally, some numerical simulations are used to prove our analytical result.

Keywords Logistic growth, Saturated treatment, Stationary distribution and ergodicity, Non-monotone incidence, Extinction.

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1. Introduction

Infectious diseases are caused by pathogens and spread through air, water, food and other vectors. In 1927, the Black Death broke out in London. Through the study of the Black Death, the classical SIR model was developed by Kermack and Mckendrick [14]. Later, they proposed the threshold theory based on the study of this model, which laid foundation for researchers who study the epidemic model. From then on, many researchers had begun to analyze epidemics through mathematical models [6, 11,21,29]. In the study of infectious diseases, mathematical models have contributed to the prediction and control of infectious diseases (see [3, 38]).

In the dynamics of infectious diseases, the SIR model divides the total population into the following three categories: Susceptible, Infected and Removed. SIR epidemic model can be used to simulate the behavior of some infectious diseases, such as HIV/AIDS, tuberculosis (TB), measles and dengue [24]. In traditional studies, the authors usually assumed that the incidence of infectious diseases is bilinear $g(I)S = \beta IS$ [28]. However, in real life, when an infectious disease infects a significant number of people, bilinear incidence is not suitable for the study of this situation. Therefore, in order to deal with different situations, many researchers have further investigated the incidence of infectious diseases. Liu, Levin and Iwasa [25] proposed the general nonlinear incidence $g(I)S = \frac{KI^pS}{1+aI^q}$, where $K, a, p, q > 0$. In

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1973, there was an outbreak of cholera in southern Italy. Capasso and Serio studied it, and found that a saturated incidence rate $g(I)S = \frac{KIS}{1+aI}$ was more suitable for studying the infectious disease of Bari, where $g(I) = \frac{KI}{1+aI}$ and KI represented the infectious ability of the disease [2,18,30,44]. We make I large enough, and then $g(I)$ tends to a saturation level. In addition, many authors have explored more varied incidences [4, 13, 34, 37, 41, 42].

COVID-19 (Corona Virus Disease 2019) has higher transmissibility than SARS (severe acute respiratory syndrome) [37, 42]. In December 2019, there was an outbreak of COVID-19 in Wuhan, China. The Chinese government has taken effective control measures such as isolating the source of infection, quarantining citizens at home and preventing the gathering of people. The outbreak has been effectively controlled. Interventions have reduced economic losses in the long run. Therefore, the control measures of the government are necessary for the face of the rampant epidemic [7]. In order to simulate this phenomenon, Xiao and Ruan proposed the following incidence [36]

$$g(I) = \frac{KI}{1+aI^2}. \tag{1.1}$$

Compared with the nonlinear incidence rate proposed by Capasso and Serio, this incidence rate can measure some psychological effects on the population. When there are many infected people in an area, the population often choose to reduce their exposure to the outside world, which leads to a decrease in the infection force of the disease [18, 44]. The specific description is shown in Figure 1 in [36].

Different from the exponential growth model, the logistic growth model focuses on the growth of population over time. At the same time, carrying capacity is also considered with limited resources. Therefore, we have found that studying the logistic growth model is more realistic [23, 43], and it is also reasonable to consider non-monotonic incidence to simulate government interference [17].

Treatment of infected individuals is indispensable for better epidemic control. In classical epidemic models, the authors often use $T(I) = \alpha I$ as a treatment function. It follows that the treatment function $T(I)$ is proportional to I. However, when the infected population is sufficiently large, many people cannot be properly treated due to limited social resources. Thus, Wang and Ruan [35] introduced a constant treatment function, and Wang [33] showed a segmented treatment function. In this paper, we will use a nonlinear treatment function as follows

$$T(I) = \frac{aI}{1+bI}.$$

It has the advantage of describing the situation, in which the treatment rate will reach the saturation value $\frac{a}{b}$ due to the lack of medical resources and treatment experience [10]. Thus, we can know that the nonlinear treatment function is more reasonable.

According to the above analysis, Ghosh et al. [10] introduced an SIR epidemic model with logistic growth, saturated treatment function as

$$\begin{cases} \mathrm{d}S = \left[rS\left(1 - \frac{S}{K}\right) - \frac{\beta SI}{1+aI^2} - \nu S\right]\mathrm{d}t, \\ \mathrm{d}I = \left[\frac{\beta SI}{1+aI^2} - (\rho + v + \gamma)I - \frac{\alpha uI}{1+buI}\right]\mathrm{d}t, \\ \mathrm{d}R = \left[\frac{\alpha uI}{1+buI} + \gamma I + \nu S - \delta R\right]\mathrm{d}t. \end{cases} \tag{1.2}$$