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Asymptotic Behavior of a Stochastic Predator-prey Model with Beddington-DeAngelis Functional Response and Lévy Jumps*

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Abstract A stochastic two-prey-one-predator model with Beddington-DeAngelis functional response and Lévy jumps is proposed and investigated in this paper. First of all, we prove the existence and uniqueness of the global positive solution, and stochastic ultimate boundedness of the solution. Next, under a simple assumption, by using Itô formula and other important inequalities, some sufficient conditions are established to ensure the extinction and persistence in the mean of the system. The results show that neither strong white noise nor Lévy noise is conducive to the persistence of the population. Finally, the theoretical results are verified by numerical simulations.

Keywords Stochastic predator-prey model, Beddington-DeAngelis functional response, Lévy jump, Extinction, Persistence.

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1. Introduction

In natural ecology, there are many complex relationships and interactions between organisms, which constitute a biological population system together. However, the interaction between predator-prey population is considered to be the most important one. Therefore, predator-prey model has become an important topic for many scholars. In addition, it is a common phenomenon in nature that predators often feed on some competing prey. So far, many scholars have carried out extensive research on deterministic two-prey-one-predator systems (see [1, 8, 9, 12, 14, 15] and other references).

Functional response has always been an important component of predator-prey dynamics. Functional responses are generally divided into two categories. One is prey-dependent functional responses, the most common of which are Holling-I, Holling-II and Holling-III, but these functional responses only consider the density of prey. The other is predator-dependent functional responses, which generally include Beddington-DeAngelis type [4, 7], Crowley-Martin type [5] and Hassell-Varley type [10]. They consider both prey density and predator density. In ecology, species

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not only compete, but also interfere with each other. Therefore, we consider using Beddington-DeAngelis functional response in this paper. In 1975, Beddington [4] and DeAngelis [7] first introduced a predator-prey model with Beddington-DeAngelis functional response

$$\begin{cases} \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = r_1x_1 - \alpha_1x_1^2 - \dfrac{c_1x_1x_2}{a_1 + a_2x_1 + a_3x_2}, \\ \dfrac{\mathrm{d}x_2}{\mathrm{d}t} = -r_2x_2 - \alpha_2x_2^2 + \dfrac{c_2x_1x_2}{a_1 + a_2x_1 + a_3x_2}, \end{cases} \tag{1.1}$$

where $\frac{c_1x_1}{a_1+a_2x_1+a_3x_2}$ becomes the Beddington-DeAngelis functional response. $x_i(i = 1, 2)$ is the size of the ith population at time t, r_1 denotes the intrinsic growth rate of the prey, r_2 denotes death rate of the predator, c_1 represents the effect of capture rate, a_1 is a saturation constant, a_2 is the effect of handing time, and a_3 represents the magnitude of interference among predators.

On one hand, in real life, the population is inevitably affected by environmental noise. Therefore, it is very necessary to study the dynamic impact of white noise on population system. So far, a large number of scholars have proposed a variety of stochastic population models (see [6,13,16,18,19,22,26–30]). Liu et al., [22] derived some conditions for species to be stochastically permanent. They also showed that the species will become extinct with probability one, if the noise is sufficiently large. Das and Samanta [6] came to a conclusion that the environmental noise plays an important role in the extinction and persistence of prey and predator populations.

On the other hand, in ecology, sudden environmental disturbances, such as tsunamis, volcanic eruptions, avian influenza and infectious diseases, also have a very important impact on the population system. However, due to the abruptness and intensity of these events, the sampling path will be discontinuous. Therefore, these phenomena cannot be accurately described by Brownian motion. In this case, some scholars (see [2,24,25,31,32]) pointed out that Lévy jump can be introduced into the model for modeling. Zhu and Li [32] dealt with a predator-prey model of Beddington-DeAngelis type functional response with Lévy jumps. They proved that the variation of Lévy jumps can affect the asymptotic property of the system.

Inspired by the above discussion and references, we assume that the intrinsic growth rate of preys and the mortality of predator are affected by Lévy noise. That is,

$$r_1\mathrm{d}t \to r_1\mathrm{d}t + \sigma_1\mathrm{d}B_1(t) + \int_{\mathbb{Y}} \gamma_1(u)\tilde{N}(\mathrm{d}t, \mathrm{d}u),$$

$$r_2\mathrm{d}t \to r_2\mathrm{d}t + \sigma_2\mathrm{d}B_2(t) + \int_{\mathbb{Y}} \gamma_2(u)\tilde{N}(\mathrm{d}t, \mathrm{d}u),$$

$$-r_3\mathrm{d}t \to -r_3\mathrm{d}t + \sigma_3\mathrm{d}B_3(t) + \int_{\mathbb{Y}} \gamma_3(u)\tilde{N}(\mathrm{d}t, \mathrm{d}u),$$

where σ_i represents the intensity of the white noise and $\sigma_i > 0$, and $B_i(t)$ $(i = 1, 2, 3)$ is independent standard Brownian motion.

According to (1.1), based on the fact that the most common system in the ecosystem is two-prey-one-predator system, we propose the following stochastic two-prey-one-predator system with the Beddington-DeAngelis functional response and