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An Arbitrary Order Reconstructed Discontinuous Approximation to Biharmonic Interface Problem

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Dedicated to the memory of Professor Zhongci Shi

Abstract. We present an arbitrary order discontinuous Galerkin finite element method for solving the biharmonic interface problem on the unfitted mesh. The approximation space is constructed by a patch reconstruction process with at most one degrees of freedom per element. The discrete problem is based on the symmetric interior penalty method and the jump conditions are weakly imposed by the Nitsche's technique. The C^2-smooth interface is allowed to intersect elements in a very general fashion and the stability near the interface is naturally ensured by the patch reconstruction. We prove the optimal a priori error estimate under the energy norm and the L^2 norm. Numerical results are provided to verify the theoretical analysis.

AMS subject classifications: 65N30

Key words: Biharmonic interface problem, patch reconstruction, discontinuous Galerkin method.

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1 Introduction

We are concerned in this paper with the biharmonic interface problem. The biharmonic operator is a fourth-order elliptic operator which is frequently seen in the thin plate bending problem and the ions transport and distribution problem [19, 33, 40]. Recently, there are many successful finite element methods proposed and applied to solve biharmonic problems, see e.g., [7, 11, 17, 25, 35]. The biharmonic interface problem arises in the context of composite materials where the physical domain is separated by an interface and the coefficient is discontinuous across this interface. Some efforts have been made to address this kind of problem [7, 9, 10, 31, 36].

The finite element methods for interface problems can be classified into two categories: body-fitted methods and unfitted methods. Body-fitted methods are constructed on a mesh aligned with the interface. This type of methods are naturally suited to deal with the discontinuity on the interface. However, generating a high-quality body-fitted mesh can be a nontrivial and time-consuming task [29, 32, 37]. In unfitted methods, the mesh is independent of the interface, and the interface can intersect elements in a very general way. As a result, unfitted methods have gained more attention for solving the interface problem.

In 2002, A. Hansbo and P. Hansbo proposed a Nitsche extended finite element method (XFEM) for solving the second-order elliptic interface problem. The idea of this method involves constructing two separated finite element spaces on both sides of the interface and using Nitsche's types of penalty to weakly impose the jump conditions. Unfitted finite element methods based on this idea are sometimes referred to as cut finite element methods (CutFEMs). Since then, these methods have been further developed and applied to a range of interface problems, including Stokes interface problems, H(div)-and H(curl)-elliptic interface problems, elasticity interface problems, and so on. We refer to [6, 14, 15, 18, 20, 24, 32] and the references therein for recent advances.

For the biharmonic interface problem, Y. Cai and et al. proposed two Nitsche-XFEMs in [9] and [10]. In [9], the authors used the so-called modified Morley finite element to approximate the solution near the interface and proved an optimal a priori error estimate under the energy norm. In [10], they derived a mixed method based on the Ciarlet–Raviart formulation with (P_2, P_2) finite element. Due to the high order of the biharmonic operator, it is hard to implement a high-order conforming space. Therefore, we aim to use the discontinuous Galerkin (DG) method to obtain the numerical solution to the biharmonic interface problem.

In this paper, we propose an arbitrary order discontinuous Galerkin CutFEM for solving the biharmonic problem with a C^2-smooth interface. The method is based on a reconstructed approximation space that is constructed by a patch reconstruction