High-Order Local Discontinuous Galerkin Method with Multi-Resolution WENO Limiter for Navier-Stokes Equations on Triangular Meshes

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Abstract. In this paper, a new multi-resolution weighted essentially non-oscillatory (MR-WENO) limiter for high-order local discontinuous Galerkin (LDG) method is designed for solving Navier-Stokes equations on triangular meshes. This MR-WENO limiter is a new extension of the finite volume MR-WENO schemes. Such new limiter uses information of the LDG solution essentially only within the troubled cell itself, to build a sequence of hierarchical L^2 projection polynomials from zeroth degree to the highest degree of the LDG method. As an example, a third-order LDG method with associated same order MR-WENO limiter has been developed in this paper, which could maintain the original order of accuracy in smooth regions and could simultaneously suppress spurious oscillations near strong shocks or contact discontinuities. The linear weights of such new MR-WENO limiter can be any positive numbers on condition that their summation is one. This is the first time that a series of different degree polynomials within the troubled cell are applied in a WENO-type fashion to modify the freedom of degrees of the LDG solutions in the troubled cell. This MR-WENO limiter is very simple to construct, and can be easily implemented to arbitrary high-order accuracy and in higher dimensions on unstructured meshes. Such spatial reconstruction methodology improves the robustness in the numerical simulation on the same compact spatial stencil of the original LDG methods on triangular meshes. Some classical

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viscous examples are given to show the good performance of this third-order LDG method with associated MR-WENO limiter.

1   Introduction

Over the past few decades, the high-order methods have received considerable attention in the CFD community because they can provide higher accuracy at a lower cost than low-order methods [1, 14, 23, 39, 40]. In addition, the report released by NASA in 2014 also mentioned the important role of high-order numerical algorithms in the next generation CFD solver [37]. Therefore, more and more scholars put their energy into the research of high-order CFD methods, and also produced many advanced algorithms, such as the finite difference or finite volume ENO/WENO schemes [3, 23, 33, 34, 36], the Hermite WENO (HWENO) scheme [18, 43], the finite difference WCNS scheme [14], the finite volume variational reconstruction scheme [41], the DG finite element method [1,4,7,17,33,35,38], the flux reconstruction (FR) or correction procedure via reconstruction (CPR) method [40], and so on. Among them, the DG method has become very popular for solving nonlinear Euler equations and Navier-Stokes equations to arbitrary order of accuracy, due to it has many excellent features, such as the ability to handle complex geometries, compactness, the flexibility of hp-refinement, and good mathematical properties for conservation, stability, and convergence, etc [4,7,15–17,24,28,29,35,38].

However, the implementation of DG methods still suffers from many limitations. In particular, how to effectively control spurious oscillations near strong discontinuities while maintaining low dissipation characteristics in smooth regions remains one of the unsolved issues in the DG methods, especially for cases above the third-order accuracy. Therefore, many scholars have devoted themselves to the method research on how to eliminate the non-physical oscillations of the DG methods in the vicinity of discontinuities. What we all know is that the pioneering work of the DG method was a series of papers proposed by Cockburn and Shu [9–11], which was developed by combining the continuous Galerkin finite element method with the idea of upwind finite volume schemes. In [8, 12, 13], they extended it to multidimensional systems on structured and unstructured meshes. In these articles, the minmod-type total variation bounded (TVB) limiter was employed to suppress the non-physical oscillations near the discontinuities, which was a slope limiter derived from the finite volume methods. Although the TVB limiter can well solve the flow problems with discontinuities, it contained artificial parameters related to the problem [45]. Therefore, the latter scholars carried out relevant research on how to design a limiter with strong universality. For example, the