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SDF-Based ILW: Inverse Lax-Wendroff Method with the Signed Distance Function Representation of the Geometric Boundary

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Abstract. This paper studies the geometric boundary representations for Inverse Lax-Wendroff (ILW) method, aiming to develop a practical computer-aided engineering method without body-fitted meshes. We propose the signed distance function (SDF) representation of the geometric boundary and design an extremely efficient algorithm for foot point calculation, which is particularly in line with the needs of ILW. Theoretical and numerical analyses demonstrate that the SDF representation of geometric boundary can satisfy ILW's needs better than others. The effectiveness and robustness of our proposed method are verified by simulating initial boundary value computational physical problems of Euler equation for compressible fluids.

AMS subject classifications: 65M06, 65M99

Key words: Computer-aided engineering, Inverse Lax-Wendroff, signed distance function, Euler equation.

1 Introduction

Computer-aided engineering (CAE) is an important part of modern industrial systems, and geometric representation plays an important role in CAE. Traditional CAE methods mostly solve differential equations of computational physical problems by using body-fitted meshes on the original computer-aided design (CAD) models as their geometric representation. However, using body-fitted meshes presents problems. Firstly, meshing a model takes a long time. Secondly, obtaining high-quality model meshes is difficult.

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Thirdly, body-fitted meshes may need a special structure, which complicates the differential equations. Isogeometric analysis (IGA, [1,2]) is proposed to deal with such problems and works well on two-dimensional problems. However, for three-dimensional problems, IGA needs volume parameterization, which will bring great difficulties.

To develop a practical CAE method without body-fitted meshes, this paper employs the Inverse Lax-Wendroff (ILW) method, which is a boundary treatment for the finite difference method (FDM) on Cartesian grids. ILW can avoid generating body-fitted meshes on CAD models. It is designed to define the values of ghost points, which are the grid points involved in simulation but outside the domain of the differential equations' solution. In [3] and [4], the authors proposed a Lax-Wendroff type boundary treatment to solve the traffic flow problem by considering an Eikonal equation in every time step. In [5], the two papers' ideas were refined, and the ILW was proposed to solve the hyperbolic conservation law equation. In the paper, the normal derivatives of the solution function at the boundary are transformed into time derivatives and tangential derivatives by differential equations and boundary conditions. Then the function values at the ghost points are calculated by Taylor expansion in the normal directions. Contrary to the idea of calculating the time derivatives with the space derivatives in the Lax-Wendroff scheme, the Inverse Lax-Wendroff method uses the differential equation to calculate space derivatives with time derivatives, to extend the equation's solution to the ghost points, which is the reason for the term Inverse.

Researchers have made many improvements to and developments of ILW. In [6], simplified ILW was proposed to reduce the calculation complexity. In [7], a survey and development of ILW can be found. In [8], ILW was used to deal with Sonic Point to avoid extremely small denominators. ILW can also be applied to solve other computational physical problems [9–14]. Some researchers analysed the conservation of ILW [15] and the stabilities of ILW [16–18]. In [19], the authors extended ILW to the moving boundary treatments, which focused on an inviscid fluid with free-slip, no-penetration, and only translational boundary conditions. Following the method in [19], an improved material derivative was introduced in [20] to extend it to rigid body motion. Recently in [21], a moving boundary ILW method was proposed to simulate 3D shock-wave impingement.

The geometry is the underpinning of ILW. No matter which version of ILW to be used and what differential equations need to be solved, ILW needs to identify whether the grid points are ghost points, calculate the foot points from the ghost points to the geometric boundary, and get the differential information of the differential equation's solution at the foot points. These tasks all need to rely on the representation of the geometry, so it is very important to find a suitable geometric representation for ILW.

The signed distance function (SDF) is a classical continuous implicit geometric boundary representation that describes the signed distance from a point to a manifold. Its sign characterizes the internal and external parts of a closed manifold, and its zero-level set implicitly represents the underlying manifold. Many popular geometric boundary representations, whether explicit forms (point cloud, mesh, or parametric representations) or implicit forms, can be easily transformed into the SDF representation [22–26]. The SDF