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Coiflet Wavelet-Homotopy Solutions to Bio-Thermal Convection in a Square Cavity

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Abstract. In this paper, the natural convection of a complex fluid that contains both nanoparticles and gyrotactic microorganisms in a heated square cavity is considered. The Buongiorno model is applied to descirbe the nanofluid behaviours. Both the top and bottom horizontal walls of the cavity are adiabatic, and there is a temperature difference between the left and right vertical walls. The non-dimensional governing equations are obtained when the stream-vorticity formulation of function is used, which are solved by the recently developed robust Coiflet wavelet homotopy analysis method. A rigid verification for the solver is given. Besides, the effects of various physics parameters including the Rayleigh number, the buoyancy ratio parameter, the bioconvection Rayleigh number, the Prandtl number, the Brownian motion parameter, the thermophoresis parameter, the heat generation parameter, the Lewis number, the bioconvection Peclet number and the Schmidt number on this complicated natural convection are examined. It is known that natural convection is closely related to our daily life owing to its wide existence in nature and engineering applications. We believe that our work will make a significant contribution to a better understanding of the natural convection of a complex fluid in a cavity with suspensions of both inorganic nanoparticles and organic microorganisms.

AMS subject classifications: 65M10, 78A48

Key words: Cavity flow, natural convection, bio-thermal convection, Coiflet wavelet, homotopy analysis method.

1 Introduction

There has been a long history of natural convection studies in cavities, many associated practical applications have been well demonstrated by different researchers [1–3]. Many

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investigators have devoted themselves to the study of behaviours of natural convection in cavities in various aspects. For example, Ruhul [4] and Ozoe et al. [5] studied the effect of surface roughness on the natural convection in an enclosure. They concluded that the presence of rough elements on the hot horizontal wall enhances the total heat transfer rate when comparing to the enclosure with smooth walls. Davis [6] developed a model to simulate natural convection of air inside a cavity with differently heated walls. His model was employed by Cheikh et al. [7] to study the influence of thermal boundary conditions on the natural convection in a square enclosure for a wide range of the Rayleigh numbers. Also his model was extended by Dixit and Babu [8] to examine the natural convection of air in a square cavity. Very recently, Khatamifar *et al.* [9] studied the heat transfer of transient conjugate natural convection in a partially heated cavity. Rao and Prabir [10] studied the natural convection in a wavy porous cavity driven by heat sources.

With the development of new materials, novel manufacturing techniques, researchers again paid their attention to explore new transport mechanisms of flow and heat transfer of the classic natural convection problems. Accordingly, new theoretical and numerical modelling were developed to attack natural convection different research areas in science and engineering. Currently, nanofluids have emerged as an important working fluid in many heat transport processes since their thermo-physical properties can feasibly tuned for different industrial and engineering purposes. The concept of nanofluids was firstly proposed by Choi and Eastman [11] in 1995, who noticed that the heat transfer performance of regular fluids is significantly enhanced as small metallic particles in nano-scale are appended. Ever since, the characters of nanofluids have been widely reported in many papers by various authors including Buongiorno [12], Hayat [13], Turkyilmazoglu [14], Chamkha [15], Dixit [16], Zargartalebi [17] and Mahdy [18] and in some books by some researchers such as Nield and Bejan [19] and Minkowycz et al. [20]. Meanwhile the natural convection of nanofluids associated with cavity flows is also invoked the interest of many scholars. For instance, Hamid et al. [21], Jou and Tzeng [22] and Oztop and Abu-Nada [23] respectively studied the natural convection of heated rectangular enclosure filled with nanofluids. Sun and Pop [24,25] investigated the natural convection in a triangular cavity. Lai et al. [26] considered the natural convection of Al_2O_3/water in a square enclosure. Tahmasebi et al. [27] studied local thermal non-equilibrium conjugate natural convection in a cavity with a porous medium filled with nanofluids. Roy [28] examined the natural convection of nanofluids in a cavity with different shapes. Rashad et al. [29] studied the MHD mixed convection flow and entropy generation of a nanofluid in a lid-driven U-shaped cavity. Zoubair et al. [30] studied the free convection and entropy generation of hybrid nanofluids inside an enclosure using the generalized Buongiorno's model. Further publications in literature on natural convection of nanofluids based on Buongiorno's model have been studied by many researchers such as Soleimani et al. [31], Rashed et al. [32], Ahmed and Xu [33], Sheremet and Pop [34,35], Ahmed and Rashed [36]. Also, several works have shown the great potential of nanofluids for practical applications in micro-systems including micro-heat pipes [37], micro-channels [38] and micro-channels heat sinks [39].