Ann. Appl. Math.
doi: 10.4208/aam.OA-2021-0082

Vol. **38**, No. 4, pp. 414-440
November 2022

Global Attractiveness and Quasi-Invariant Sets of Impulsive Neutral Stochastic Functional Differential Equations Driven by Tempered Fractional Brownian Motion

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Received 19 October 2021; Accepted (in revised version) 30 September 2022

Abstract. In this paper, we are concerned with a class of impulsive neutral stochastic functional different equations driven by tempered fractional Brownian motion in the Hilbert space. We obtain the global attracting and quasi-invariant sets of the considered equations driven by tempered fractional Brownian motion $B^{\alpha,\lambda}(t)$ with $0<\alpha<1/2$ and $\lambda>0$. In particular, we give some sufficient conditions which ensure the exponential decay in the p-th moment of the mild solution of the considered equations. Finally, an example is given to illustrate the feasibility and effectiveness of the results obtained.

AMS subject classifications: 60H15

Key words: Global attracting set, quasi-invariant sets, tempered fractional Brownian motion, exponential decay.

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1 Introduction

In the past twenty years, the fractional Brownian motion (fBm in short) $B^H(t)$ has attracted the increasing attention due to its wide applications in mathematical finance (see [1]); in biology (see [2]); in communication networks (see, for instance [3]); the analysis of global temperature anomaly [4] and electricity markets [5] etc. Many

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interesting works have been done on SDEs driven by fBm and ones can refer to [6–10]. In addition, impulsive neutral stochastic functional differential equations driven by fractional Brownian motion have been widely investigated in the literature, see, e.g., [11–14] and the references therein.

Tempered fractional Brownian motion (tfBm) $B^{\alpha,\lambda}(t)$ was first introduced by Meerschaert and Skarabzi in [15]. It was defined by exponentially tempering the power law kernel in the moving average representation of a fractional Brownian motion. The tempered fractional Gaussian noise (TFGN), the increment of the tempered fractional Brownian motion, forms a stable time series that can exhibit semi-long range dependence. In addition, TFGN can provide useful stochastic process models for wind speed data, see, e.g., [16–18]. And the time-changed tfBm has been investigated in [19] with potential applications in financial time series, biology and physics. When $\lambda=0$ and $-1/2<\alpha<1/2$, the tfBm reduces a fractional Brownian motion, a self-similar Gaussion sochastic process with Hurst scaling index $H=1/2-\alpha$.

On the other hand, attracting sets of stochastic dynamical systems have been extensively studied over the last a few decades due to weakening the stability conditions of stochastic system. For example, Li and Xu [20] investigate the global attracting set, exponential stability and stability in distribution of SPDEs with jumps. Liu and Li [21] gave the gobal attracting sets and the conditions of exponential stability in distribution of neutral SPDEs driven by α-stable processes. Recently, Wang and Liu [22] investigate the exponential behavior and upper noise excitation index of solutions to evolution equations with unbounded delay and tfBm with $-1/2<\alpha<0$ and $\lambda>0$. As for as I known, few articles introduce impulse neutral stochastic differential equations driven by tfBm. Thus, the purpose of this paper is to obtain the global attractiveness and quasi-invariant sets of impulsive neutral stochastic functional differential equations driven by tfBm with $0<\alpha<1/2$ and $\lambda>0$.

In this paper, we consider the impulsive neutral stochastic functional differential equations driven by tfBm

$$\begin{cases} d[u(t)+G(t,x_t)]=[Ax(t)+f(t,x_t)]dt+g(t,x_t)d\omega(t) & \\ \qquad\qquad\qquad\qquad +\sigma(t)dB^{\alpha,\lambda}(t), & t\geq 0, \quad t\neq t_k, \\ \Delta x(t_k)=x\left(t_k^+\right)-x\left(t_k^-\right)=I_k(x(t_k)), & t=t_k, \quad k=1,2,\cdots, \\ x(t)=\varphi(t)=PC^B_{\mathcal{F}_0}((-\tau,0];X), & t\in[-\tau,0], \end{cases} \tag{1.1}$$

where $B^{\alpha,\lambda}(t)$ is the tempered fractional Brownian motion with $0<\alpha<1/2$ and $\lambda>0$. A is the infinitesimal generator of an analytic semigroup $S(t)$, $t\geq 0$, on the separable Hilbert space X. ω is a standard Wiener process on a real and separable Hilbert space Y. Further, we assume that ω and $B^{\alpha,\lambda}(t)$ are independent; f, G: $\mathrm{R}^+\times X\to X$. g: $\mathrm{R}^+\times X\to\mathcal{L}_2^0(Y,X)$ and σ: $\mathrm{R}^+\to\mathcal{L}_Q^0(Y,X)$ are jointly continuous functions; the