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Asymptotic Behavior of Solutions for the Porous Media Equations with Nonlinear Norm-type Sources

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Abstract. In the paper, the asymptotic behavior of the solution for the parabolic equation system of porous media coupled by three variables and with weighted nonlocal boundaries and nonlinear internal sources is studied. by constructing the upper and lower solutions with the ordinary differential equation as well as introducing the comparison theorem, the global existence and finite time blow-up of the solution of parabolic equations of porous media coupled by the power function and the logarithm function are obtained. The differential inequality technique is used to obtain the lower bounds on the blow up time of the above equations under Dirichlet and Neumann boundary conditions.

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Key Words: Porous media equations; norm-type sources; the global existence; the finite time blow-up; the blow up time.

1 Introduction

In this article, we consider the following nonlocal cross-coupled porous medium equations

$$\begin{cases} u_t=\Delta u^{m_1}+a\|u^{p_1}[ln(v+1)]^{q_1}\|_{\alpha}^{r_1}, & x\in\Omega,\ t>0,\\ v_t=\Delta v^{m_2}+b\|v^{p_2}[ln(w+1)]^{q_2}\|_{\beta}^{r_2}, & x\in\Omega,\ t>0,\\ w_t=\Delta w^{m_3}+c\|w^{p_3}[ln(u+1)]^{q_3}\|_{\gamma}^{r_3}, & x\in\Omega,\ t>0, \end{cases} \tag{1.1}$$

and with continuous bounded initial values,

$$u(x,0)=u_0(x),\ v(x,0)=v_0(x),\ w(x,0)=w_0(x),\quad x\in\Omega, \tag{1.2}$$

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when the equations has weighted nonlocal boundary flow,

$$\begin{cases} u(x,t)=\int_\Omega \phi_1(x,y)u(y,t)\mathrm{d}y, & x\in\partial\Omega,\ t>0, \\ v(x,t)=\int_\Omega \phi_2(x,y)v(y,t)\mathrm{d}y, & x\in\partial\Omega,\ t>0, \\ w(x,t)=\int_\Omega \phi_3(x,y)w(y,t)\mathrm{d}y, & x\in\partial\Omega,\ t>0, \end{cases} \tag{1.3}$$

sufficient conditions for global existence and blow up of solutions in finite time are obtained. When the equations have Dirichlet boundary conditions,

$$u(x,t)=v(x,t)=w(x,t)=0, \qquad (x,t)\in\partial\Omega\times(0,t^\star), \tag{1.4}$$

or Neumann boundary conditions,

$$\frac{\partial u^{m_1}}{\partial v}=lu,\ \frac{\partial v^{m_2}}{\partial v}=lv,\ \frac{\partial w^{m_3}}{\partial v}=lw, \qquad (x,t)\in\partial\Omega\times(0,t^\star), \tag{1.5}$$

the lower bounds of blow up time for solutions are obtained.

Where $\Omega\in R^N(N>1)$ is a bounded region with a smooth boundary, $m_i>1$, $p_i>1$, $q_i>1$, $r_1>\alpha$, $r_2>\beta$, $r_3>\gamma>1$, $a,b,c>0(i=1,2,3)$, the weight function $\phi_i(x,y)(i=1,2,3)$ is a non-negative continuous function in $\partial\Omega\times\Omega$, which satisfies $0<\int_\Omega\phi_i(x,y)\mathrm{d}y\le 1$. The initial values $u_0,v_0,w_0\in C^{2+h}(\overline{\Omega})(0<h<1)$ are non-negative and satisfies the compatibility condition on the boundary, v is the unit external normal vector in the external normal direction of $\partial\Omega$, The norm here is

$$\begin{cases} \|u^{p_1}[ln(v+1)]^{q_1}\|_\alpha^{r_1}=\left[\int_\Omega (u^{p_1}(ln(v+1))^{q_1})^\alpha \mathrm{d}x\right]^{\frac{r_1}{\alpha}}, \\ \|v^{p_2}[ln(w+1)]^{q_2}\|_\beta^{r_2}=\left[\int_\Omega (v^{p_2}(ln(w+1))^{q_2})^\beta \mathrm{d}x\right]^{\frac{r_2}{\beta}}, \\ \|w^{p_3}[ln(u+1)]^{q_3}\|_\gamma^{r_3}=\left[\int_\Omega (w^{p_3}(ln(u+1))^{q_3})^\gamma \mathrm{d}x\right]^{\frac{r_3}{\gamma}}. \end{cases} \tag{1.6}$$

The equations can be used to describe the reaction-diffusion problems of the three kinds of media in physics, such as problems of porous medium mechanics, fluid mechanics, and gas flow, which have been studied extensively by many scholars [1-4].

Deng in [5] studied the following norm source equations

$$u_t=\Delta u^m+a\|v\|_\alpha^p, \quad v_t=\Delta v^n+b\|u\|_\beta^q, \quad x\in\Omega,\ t>0,$$

under homogeneous Dirichlet boundary conditions, when $pq<mn$, every nonnegative solution global existence, when $pq>mn$, the global solution and the blow up solution exist simultaneously, and $p_c=pq-mn$ is the critical quota of the above equations.