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# The Upper Semicontinuity of Random Attractor for Stochastic Suspension Bridge Equation*

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**Abstract** Based on the existence of pullback attractors for stochastic suspension bridge in [7], in the paper, we further investigate the upper semicontinuity of pullback attractors for the problem.

**Keywords** Upper semicontinuity, Suspension bridge equations, Random attractors, Linear memory, Additive noise.

**MSC(2010)** 35B40, 35B41.

## 1. Introduction

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, where

$$\Omega = \{\omega = (\omega_1, \omega_2, \cdots, \omega_m) \in C(\mathbb{R}, \mathbb{R}^m) : \omega(0) = 0\},$$

is endowed with compact open topology, $\mathcal{F}$ is the $\mathbb{P}$-completion of Borel $\sigma$-algebra on $\Omega$, and $\mathbb{P}$ is the corresponding Wiener measure. Define the time shift via

$$\theta_t\omega(\cdot) = \omega(\cdot + t) - \omega(t), \quad t \in \mathbb{R}, \ \omega \in \Omega.$$

Thus, $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ is an ergodic metric dynamical system.

In this paper, we are devoted to considering the upper semicontinuity of random attractors for the following suspension bridge equations with linear memory and additive white noise:

$$\begin{cases} u_{tt} + \Delta^2 u + \Delta^2 u_t + ku^+ + (p - \beta\|\nabla u\|^2_{L^2(U)})\Delta u + \int_0^\infty \mu(s)\Delta^2(u(t) \\ \qquad - u(t-s))ds = g(x) + \alpha \sum_{j=1}^m h_j \dot{W}_j, \ x \in U, t > \tau, \\ u(x,t) = \Delta u(x,t) = 0, \qquad x \in \partial U, \ t \le \tau, \ \tau \in \mathbb{R}, \\ u(x,\tau) = u_0(x), u_t(x,\tau) = u_1(x), \qquad x \in U, \end{cases} \tag{1.1}$$

where $U$ is a bounded open set of $\mathbb{R}^2$ with a smooth boundary $\partial U$, $u = u(x,t)$ is a real-valued function on $U \times [\tau, +\infty)$ and accounts for the downward deflection of

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the bridge in the vertical plane, $u^+$ namely stands for its positive part,

$$u^+ = \begin{cases} u, & \text{if } u \geq 0, \\ 0, & \text{if } u \leq 0. \end{cases}$$

$k > 0$ denotes the spring constant, and $\alpha$ is a positive constant. The real constant $p$ represents the axial force acting at the end of the road bed of the bridge in the reference configuration. Namely, $p$ is negative when the bridge is stretched, positive when compressed, $h_j(x) \in H_0^2(U) \cap H^4(U)$, $(j = 1, 2, 3, \cdots, m)$, $\{W_j\}_{j=1}^m$ are independent two-sided real-valued Wiener processes on $(\Omega, \mathfrak{F}, \mathbb{P})$. We identify $\omega(t)$ with $(W_1(t), W_2(t), \cdots, W_m(t))$, i. e.,

$$\omega(t) = (W_1(t), W_2(t), \cdots, W_m(t)), t \in \mathbb{R}.$$

The memory kernel function $\mu(s)$ and $g(x)$ satisfy the following conditions:

$(H_1)$ : $\mu(s) \in C^1(\mathbb{R}^+) \cap L^2(\mathbb{R}^+)$, $\mu(s) \geq 0$, $\mu'(s) + \delta\mu(s) \leq 0$, $\forall s \in \mathbb{R}^+$ and some $\delta > 0$.

$(H_2)$ : $g \in H_0^1 \cap H^2(U)$.

Following Dafermos [1], we introduce a Hilbert "history" space

$$\mathfrak{R}_{\mu,2} = L^2_\mu(\mathbb{R}^+, H^2(U) \cap H_0^1(U))$$

with the inner product

$$(\eta_1, \eta_2)_{\mu,2} = \int_0^\infty \mu(s) \left(\Delta\eta_1(s), \Delta\eta_2(s)\right) ds, \quad \forall \eta_1, \eta_2 \in \mathfrak{R}_{\mu,2},$$

and new variables

$$\eta(t, x, s) = u(t, x) - u(t - s, x).$$

To facilitate easy calculation, we take $\beta = 1$. Then, we set $E = (H^2(U) \cap H_0^1(U)) \times L^2(U) \times \mathfrak{R}_{\mu,2}$, $Z = (u, u_t, \eta)^T$. Then, the system (1.1) is equivalent to the following initial value problem in the Hilbert space $E$ :

$$\begin{cases} Z_t = L(Z) + N(Z, t, W(t)), & x \in U,\ t \geq \tau,\ s \in \mathbb{R}^+, \\ Z(\tau) = Z_\tau = (u_0(x), u_1(x), \eta_0(x, s)), & (x, s) \in U \times \mathbb{R}^+, \end{cases} \tag{1.2}$$

where

$$\begin{cases} u(t, \tau, x) = \eta(t, \tau, x, s) = \eta(t, \tau, x, 0) = 0,\ x \in \partial U,\ t > \tau,\ s \in \mathbb{R}^+, \\ \Delta u(t, \tau, x) = \Delta\eta(t, \tau, x, s) = \Delta\eta(t, \tau, x, 0) = 0,\ x \in \partial U,\ t \leq \tau,\ s \in \mathbb{R}^+, \\ u(\tau, x) = u(\tau, \tau, x) = u_0(\tau, x),\ u_t(\tau, x) = u_t(\tau, \tau, x) = u_1(x), \quad x \in U, \\ \eta(\tau, x, s) = \eta_0(x, s) = u(\tau, x) - u(\tau - s, x), \quad (x, s) \in U \times \mathbb{R}^+, \end{cases} \tag{1.3}$$

$$L(Z) = \begin{pmatrix} u_t \\ -\Delta^2 u - \Delta^2 u_t - \int_0^\infty \mu(s)\Delta^2\eta(s)ds \\ u_t - \eta_s \end{pmatrix}, \tag{1.4}$$