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Existence and Uniqueness of Solutions to Time-delays Stochastic Fractional Differential Equations with Non-Lipschitz Coefficients

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Abstract In this paper, we consider the existence and uniqueness of solutions to time-varying delays stochastic fractional differential equations (SFDEs) with non-Lipschitz coefficients. By using fractional calculus and stochastic analysis, we can obtain the existence result of solutions for stochastic fractional differential equations.

Keywords Existence and uniqueness, Stochastic fractional differential equations, Time-varying delays, Non-Lipschitz coefficients.

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1. Introduction

In recent years, fractional differential equation has been a growing field of research because of their widespread applications in many real life problems, such as image processing, biology, economics, fitting of experimental data and control theory. Classical theory and applications of fractional differential equations are presented in the monographs (see [9, 10, 15, 17, 19]). Stochastic differential equations have become an active area of investigation due to their applications in finance markets, biology, telecommunications networks and other fields [2, 5, 6, 13, 16]. Further, many authors studied the existence, uniqueness, stability and controllability of solutions for stochastic differential equations by using stochastic analysis theory and fixed point theorem in related literature, see [3, 7, 8, 20–22].

In the paper [1], Abouagwa et al. consider stochastic fractional differential equations of Itô-Doob type in the following form:

$$\begin{cases} dx(t) = b\big(t, x(t)\big)dt + \sigma_1\big(t, x(t)\big)dB(t) + \sigma_2\big(t, z(x)\big)(dt)^\alpha, \ t \in [0, T], \\ x(t) = x_0 \in \mathbb{R}^n, \end{cases}$$

where $\frac{1}{2} < \alpha < 1$, T denotes a positive real number, $b : [0, T] \times \mathbb{R}^n \to \mathbb{R}^n$, $\sigma_1 : [0, T] \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$ and $\sigma_2 : [0, T] \times \mathbb{R}^n \to \mathbb{R}^n$ are measurable continuous functions. $B(t)$ is Brownian motion defined on the filtered probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P}\}$. Approximation properties for solutions to equations were established by fractional calculus, stochastic analysis, elementary inequalities and so on.

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Luo et al. [11] investigate a class of stochastic fractional differential equations with time-delays

$$\begin{cases} dx(t) = b\big(t, x(t), x(t-\tau)\big)dt + \sigma_1\big(t, x(t), x(t-\tau)\big)dB(t) \\ \qquad\qquad + \sigma_2\big(t, x(t), x(t-\tau)\big)(dt)^\alpha,\ t \in J, \\ x(\theta) = \Phi(\theta),\ \theta \in [-\tau, 0] \end{cases}$$

where $J = [0, T]$, $\frac{1}{2} < \alpha < 1$, $b : [0, T] \times \mathbb{R}^n \to \mathbb{R}^n$, $\sigma_1 : [0, T] \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$ and $\sigma_2 : [0, T] \times \mathbb{R}^n \to \mathbb{R}^n$ are measurable continuous functions. $B(t)$ is Brownian motion defined on the filtered probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P}\}$. $\phi : [-\tau, 0] \to \mathbb{R}^n$ is a continuous function satisfying $\mathbb{E}|\phi(\theta)|^2 < \infty$. Under some assumptions, the author obtained an averaging principle for the solution of the considered equations.

Motivated by the discussion above, we are concerned with the following time-varying delays in stochastic fractional differential equations

$$\begin{cases} dz(t) = b\big(t, z(t), z(t-\delta(t))\big)dt + \sigma_1\big(t, z(t), z(t-\delta(t))\big)dB(t) \\ \qquad\qquad + \sigma_2\big(t, z(t), z(t-\delta(t))\big)(dt)^\alpha,\ t \in J, \\ z(t) = \phi(t),\ t \in [-\delta, 0], \end{cases} \tag{1.1}$$

where $\frac{1}{2} < \alpha < 1$, $J = [-\delta, T]$, T denotes a positive real number, $0 \leq \delta(t) \leq \delta$, $b : J \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, $\sigma_1 : J \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$, $\sigma_2 : J \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ are measurable continuous functions, $B(t)$ is Brownian motion defined on the filtered probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P}\}$. $\phi : [-\delta, 0] \to \mathbb{R}^n$ is a continuous function satisfying $\mathbb{E}|\phi(t)|^2 < \infty$. $x(t)$ denotes a n-dimensional random variable.

2. Preliminaries

For the sake of smooth follow-up work, we briefly give the preparatory work in this section.

Definition 2.1. (Definition 2.1, [1]) For any $\alpha \in (0, 1)$ and a function $f \in L^1[[0, T]; \mathbb{R}^n]$, the Riemann-Liouville fractional integral operator of order α is defined for all $0 < t < T$ by

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s)ds,\ t > 0,$$

where $\Gamma(\cdot)$ is the Gamma function.

Lemma 2.1. *(Lemma 2.1, [1]) Let $f(t)$ be a continuous function, then its integration with respect to $(dt)^\alpha$, $0 < \alpha \leq 1$ is defined by*

$$\int_0^t f(s)(ds)^\alpha = \alpha \int_0^t (t-s)^{\alpha-1} f(s)ds,\ 1 \geq \alpha > 0.$$

Definition 2.2. An R^n-value stochastic process $x(t)_{-\tau\leq t\leq T}$ is called a unique solution to SFDEs (1.1) if $x(t)$ satisfies the following:
(i) $x(t)$ a continuous process of adaptation;