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Bifurcation Difference Induced by Different Discrete Methods in a Discrete Predator-prey Model*

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Abstract In this paper, we revisit a discrete predator-prey model with Allee effect and Holling type-I functional response. The most important is for us to find the bifurcation difference: a flip bifurcation occurring at the fixed point E_3 in the known results cannot happen in our results. The reason leading to this kind of difference is the different discrete method. In order to demonstrate this, we first simplify corresponding continuous predator-prey model. Then, we apply a different discretization method to this new continuous model to derive a new discrete model. Next, we consider the dynamics of this new discrete model in details. By using a key lemma, the existence and local stability of nonnegative fixed points E_0, E_1, E_2 and E_3 are completely studied. By employing the Center Manifold Theorem and bifurcation theory, the conditions for the occurrences of Neimark-Sacker bifurcation and transcritical bifurcation are obtained. Our results complete the corresponding ones in a known literature. Numerical simulations are also given to verify the existence of Neimark-Sacker bifurcation.

Keywords Discrete predator-prey model with Holling type-I funcational response, Flip bifurcation, Neimark-Sacker bifurcation, Transcritical bifurcation, Allee effect.

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1. Introduction and preliminaries

In the past few years, the predator-prey models have been widely studied. For a generalized predator-prey system

$$\begin{cases} \dot{x} = xp(x) - g(x)y, \\ \dot{y} = y(rg(x) - q(y)), \end{cases} \tag{1.1}$$

where x and y indicate the density of prey and predator respectively, $p(x)$ represents the growth rate of prey with the absence of predator, $q(y)$ denotes the death rate of

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predator, r is predator's efficiency rate in predating and $g(x)$ describes the predator functional response. For different predator-prey systems, Holling [7] introduced three kinds of functional response, namely, $g(x) = x$, $\frac{x}{m+x}$, $\frac{x^2}{m+x^2}$, which are called Holling type I, II and III respectively.

In order to get a more realistic model, one often considers the Allee effect for a given model. Allee effect [3], introduced by ecologist W. C. Allee, is a fundamental phenomenon in the biological system, which describes a positive relationship between the population density and per capita growth rate of the population at low densities. The Allee effect can be divided into two kinds: strong Allee effect and weak Allee effect. A critical threshold is proposed for the strong Allee effect, under which the per capita growth rate of population is negative below the threshold and the growth rate becomes positive above the threshold, while the pre capita growth rate remain positive at a low population density for the weak Allee effect. Lots of researches about predator-prey models are done with the Allee effect [1,19,21,23].

Generally speaking, it is impossible to solve a complicate system of ordinary differential equations. Therefore, one often solves its discrete version by using computer. Due to the realistic meanings of discrete models, more and more studies have applied the theory of discrete dynamical system [5,8,14,15,17].

Zhang et. al [22] first considered a Lotka-Volterra [11,16] type predator-prey system with Holling type-I functional response as follows:

$$\begin{cases} \dot{x}(t) = r_0 x(1 - \frac{x}{k}) - axy, \\ \dot{y}(t) = bxy - dy, \end{cases} \tag{1.2}$$

where x is the prey population and y is the predator population, r_0 is the intrinsic growth rate of prey, k is the carrying capacity of the environment for prey, a is the prey capture rate by their predators, b is the conversion efficiency from prey to predator and d is the intrinsic death rate of predator. The initial values of system (1.2) satisfy $x(0) > 0$, $y(0) > 0$ and all the parameters are positive.

Then, they introduced the strong Allee effect for the prey into system (1.2), and rewrite system (1.2) as

$$\begin{cases} \dot{x}(t) = r_0 x(1 - \frac{x}{k})(x - c) - axy, \\ \dot{y}(t) = bxy - dy. \end{cases} \tag{1.3}$$

Finally, the authors employed the forward Euler method to get the discrete form of system (1.3) in the following

$$\begin{cases} x_{n+1} = x_n + r_0 x_n(1 - \frac{x_n}{k})(x_n - c) - a x_n y_n, \\ y_{n+1} = y_n + b x_n y_n - d y_n. \end{cases} \tag{1.4}$$

Although the authors of [22] obtained some good results for system (1.4), some problems still exist. On one hand, when considering the dynamical properties of a given system of ordinary differential equations or differential equations, one hopes to study its equivalent simple form with as less parameters and variables as possible. System (1.3) has 6 parameters, and is not a simple form. In fact, by letting $\frac{x}{k} \to x$,