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Mathematical Modeling and Analysis of an Epidemic Model with Quarantine, Latent and Media Coverage

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Abstract Epidemic models are very important in today's analysis of diseases. In this paper, we propose and analyze an epidemic model incorporating quarantine, latent, media coverage and time delay. We analyze the local stability of either the disease-free and endemic equilibrium in terms of the basic reproduction number $\mathcal{R}_0$ as a threshold parameter. We prove that if $\mathcal{R}_0 < 1$, the time delay in media coverage can not affect the stability of the disease-free equilibrium and if $\mathcal{R}_0 > 1$, the model has at least one positive endemic equilibrium, the stability will be affected by the time delay and some conditions for Hopf bifurcation around infected equilibrium to occur are obtained by using the time delay as a bifurcation parameter. We illustrate our results by some numerical simulations such that we show that a proper application of quarantine plays a critical role in the clearance of the disease, and therefore a direct contact between people plays a critical role in the transmission of the disease.

Keywords Epidemic model, Stability, Hopf bifurcation, Delay.

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1. Introduction

For a long time, infectious and epidemics have been a great challenge to mankind, and people have tried every means to prevent and control them. Therefore, the analysis of the dynamics of diseases has always been a hot topic in people's research. At the same time, the frequent occurrence of HIV (see [34,38–41]), tuberculosis (TB) (see [19, 20, 29]), swine flu (see [26, 47, 49, 56]), Avian flu (see [15, 24, 27, 33]), Ebola (see [6, 16, 23, 59]), human influenza (see [4, 27, 58]), Zika virus (see [2, 3, 10, 11]), severe acute respiratory syndrome (SARS) (see [5, 7, 14]) and COVID-19 (see [12, 30–32, 50, 52]) in recent years has made people more aware of the importance of studying the prevention and transmission mechanism of diseases.

Many scholars rely on the transmission mechanism and impact of infectious and epidemics diseases factors from the perspective of reality, establishing a reasonable

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infectious disease model, and through the specific analysis of each parameter in the model, the specific measures should be taken so as to theoretically provide a strong basis for the prevention and control of the occurrence and spread of disease. In addition, people's sense of self-protection has a positive effect on the prevention and control of diseases.

In many parts of the world, the mass media plays an irreplaceable role of changing public health-related behaviour. For example, in the early stage of a disease outbreak, the government and the Centers for Disease Control (CDC) can let people know the harm of the disease and related prevention measures through various media in a timely manner so as to minimize the chance of catching disease and achieve the purpose of curbing the epidemic. Therefore, it is particularly important to consider the infectious disease models incorporating media impact. Media reports can make the behavior of susceptible populations dependent on a continuous change in the number of cases, which is reflected by a continuous differentiable infection rate function. In [44], Liu et al. introduced an EIH epidemic model with media effects. The function of impact factor is $f(E,I,H) = e^{-a_1E-a_1I-a_3H}$, where E, I, H are the exposed, infectious and hospitalized individuals, and a_1, a_2, a_3 are non-negative parameters to measure the effect of psychological impact of media reported numbers of exposed, infectious and hospitalized individuals. The rationality of the impact factor in the model was tested by simulating the outbreak of SARS in the greater Toronto area in 2003. In [43], Liu et al. constructed a novel saturation disease rate function $\beta(I) = \beta_1 - \frac{\beta_2 I}{m+I}$ to describe the intrinsic property of the continuous change in the number of cases reported by media in public behavior, which reduces the population exposure rate to a certain limited level, where β_1 is the contact rate before media alert, and β_2 is the maximal reduced effective contact rate due to mass media alert in the presence of infective individuals. The term $\frac{\beta_2 I}{m+I}$ measures the effect of reduction of the contact rate when infectious individuals are reported in the media (see [19, 42–46, 58, 61]). The function $g(I) = \frac{I}{m+I}$ is continuous bounded function which takes into account disease saturation or psychological effects (see [13, 58, 61]). Because the coverage report cannot prevent disease from spreading completely we have $\beta_1 \geq \beta_2 > 0$. The half saturation constant $m > 0$ reflects the impact of media coverage on the contact transmission ([19, 42–46, 58, 61]). They propose and analyze a mathematical model of tuberculosis (TB) transmission considering social awareness effects during an epidemic. Das et al. [19] proposed and analyzed a mathematical model of tuberculosis (TB) transmission considering social awareness effects during an epidemic. To quantify the effect of media awareness in disease transmission rate, they updated the transmission coefficient β_1 both in susceptible and exposed class by $\beta^a(I) = \beta_1 - \frac{\beta_2 I}{m+I}$ and $\beta^b(I) = \beta_1 - \frac{\beta_3 I}{m+I}$ respectively. Here, $\beta_1 > \beta_2, \beta_3$. In general, there are two approaches to account the effect of media awareness: (i) updating the disease transmission rate to accumulate the significant fall in transmission due to preventive measures (see [19, 43, 45, 46, 58]) and (ii) by incorporating a mass media compartment to represent the public interaction with mass media ([17, 36, 60]).

On the other hand, the delays are in the media coverage. Media coverage of an disease outbreak can be seen as following two major routes [5, 57, 61]. The first route is when the media report directly to the public about the facts, and the second has public health authorities that use the media or the Internet to inform about an outbreak. For the second route, the number of infections and the number of suspected infections reported by media today are often the statistical