

# Investigating the Turing Conditions for Diffusion-driven Instability in Predator-prey System with Hunting Cooperation Functional Response

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**Abstract** In this paper, we focus on stability analysis of steady-state solutions of a predator-prey system with hunting cooperation functional response. The results show that the Turing instability can be affected not only the existence of hunting cooperation, but also the diffusion coefficients: (1) in the absence of predator diffusion, diffusion-driven instability can be induced by hunting cooperation, but no stable patterns appear; (2) the system can occur diffusion-driven instability and Turing patterns, when both predator and prey have diffusion, and the diffusion coefficient of prey is greater than that of the predator. The numerical simulations of two cases are presented to verify the validity of our theoretical results.

**Keywords** Reaction-diffusion-ordinary differential equations, Steady state solutions, Hunting cooperation functional response, Carnivorous plants and insects model.

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## 1. Introduction

Predator-prey systems are classical mathematical models of biology, and have been widely concerned. In the dynamics of interacting predator-prey populations, the functional response is an important aspect in determining the different dynamical behaviors. Functional response is the number of prey successfully attacked per predator, and it represents the attack ability of the predator to the prey (see [4, 19, 21, 22, 32]). Hunting cooperation is a common factor in the predator-prey system. In fact, a direct consequence of hunting cooperation observed in many species (predators, birds, aquatic creatures, spiders) is the “Allee effect” on predators, that is, predators can continue to survive in the absence of hunting cooperation even if prey population is insufficient to sustain them. In [1, 3, 6, 11, 13, 16–18, 27, 28, 31], the authors showed that hunting cooperation can be beneficial to the predator population by increasing the attack rate. For example, Alves and Hilker [1] added a cooperation term to the attack rate of the predator population, and proposed the

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following functional response

$$\Phi(u, v) = (\lambda + av) u,$$

where  $\lambda > 0$  is the attack rate of the per predator on the prey and  $a \geq 0$  describes the predator cooperation in hunting. Then, the corresponding predator-prey model is

$$\begin{cases} \frac{du}{dt} = ru \left(1 - \frac{u}{K}\right) - (\lambda + av) uv, \\ \frac{dv}{dt} = ev(\lambda + av) - mv, \end{cases} \quad (1.1)$$

where  $u$  and  $v$  are prey and predator densities respectively,  $r$  is the per capita intrinsic growth rate of prey,  $K$  is the carrying capacity of prey,  $e$  is the conversion efficiency, and  $m$  is the per capita mortality rate of predators. All the parameters involved in system (1.1) are assumed to be positive from the viewpoint of ecology. In [17], the authors introduced the self-diffusion and cross-diffusion into (1.1), and considered the following system

$$\begin{cases} \frac{\partial u}{\partial t} = d_{11}\Delta u + ru \left(1 - \frac{u}{K}\right) - (\lambda + av) uv, \\ \frac{\partial v}{\partial t} = d_{22}\Delta v + ev(\lambda + av) - mv, \end{cases} \quad (1.2)$$

where the nonnegative constants  $d_{11} \geq 0$  and  $d_{22} \geq 0$  are the self-diffusion coefficients of the prey and predator populations.  $\Delta$  is the usual Laplacian operator. Under the condition  $0 \leq d_{11} \leq d_{22}$ , there is no diffusion-driven Turing instability and the self-diffusion does not induce the Turing instability (see [17]).

Considering the predator-prey relationship between carnivorous plants and insects, scientists have found that carnivorous plants can benefit from insect capture through increased growth, earlier flowering and increased seed production. The ability of plants to feed on animals evolved independently in many plant lineages. The carnivorous habit has been described in about 600 species of angiosperms. If diffusion is introduced into the predator-prey relationship between carnivorous plants and insects, the diffusion rate of prey is much faster than that of predators. Then, we have

$$0 \leq d_{22} \leq d_{11}.$$

In this paper, we will investigate two cases: (1)  $d_{22} = 0$ ; (2)  $0 < d_{22} < d_{11}$ . It is clear that in case (1), (1.2) is a system of a single reaction-diffusion equation coupled with the ordinary differential equation which has very different properties with the classical reaction-diffusion equation. Many researchers in this field produced impressive results (see [2, 5, 8–10, 23, 25]). As a case, the authors in [9] pointed out that the reaction-diffusion-ODE model exhibited diffusion-driven instability (Turing instability) under a condition of autocatalysis of non-diffusing components. However, the same mechanism, which destabilized constant solutions of such models, also destabilized all continuous spatially heterogeneous stationary solutions. As a result, there existed no stable Turing patterns in such reaction-diffusion-ODE systems. A reaction-diffusion equation coupled to an ordinary differential equation on convex domains was considered in [23]. However, under general cooperative or competitive interactions, the results did not support interesting patterns. To investigate the effect of diffusion on patterns, we also consider the case  $d_{22} \neq 0$ . There are many