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Hopf Bifurcation Analysis of a Host-generalist Parasitoid Model with Diffusion Term and Time Delay*

Zijun Liu[1] and Ruizhi Yang[1,†]

Abstract In this paper, we studied a delayed host-generalist parasitoid model with Holling II functional response and diffusion term. The Turing instability and local stability are studied. The existence of Hopf bifurcation is investigated, and some explicit formulas for determining the bifurcation direction and the stability of the bifurcating periodic solution are derived by the theory of center manifold and normal form method. Some numerical simulations are carried out.

Keywords Delay, Diffusion, Turing instability, Hopf bifurcation.

MSC(2010) 34K18, 35B32.

1. Introduction

In many aspects, dynamics of population model has been studied [1,4,5,16]. Host-generalist parasitoids systems have gotten great attention in recent years. Because of the invasion of leafmicrolepidopteron attacking horse chestnut trees in Europe (in particular in France) [8], Magal et al. [3] investigated the following host-parasitoid model with Holling Type II functional response, that is

$$\begin{cases} \frac{du(t)}{dt} = r_1 u - \frac{r_1 u^2}{K_1} - \frac{\xi uv}{1+\xi hu} \\ \frac{dv(t)}{dt} = r_2 v - \frac{r_2 v^2}{K_1} + \frac{\gamma \xi uv}{1+\xi hu}, \end{cases} \tag{1.1}$$

where $u(t)$ and $v(t)$ denote densities of the hosts(leafminers Cameraria orhidella) and generalist parasitoids (Minotetrastichus frontalis) at time t respectively. r_1 is the intrinsic growth rate of the hosts in absence of parasitoids. r_2 represents the intrinsic growth rate of the parasitoids in absence of hosts. K_1 denotes the carrying capacity of the host population. K_2 denotes the carrying capacity of the parasitoid population. ξ is the encounter rate of hosts and parasitoids. γ is the conversion rate of parasitoids. h describes the harvesting time. r_i, $K_i (i = 1, 2)$, γ, ξ, h are all positive constants. Magal et al. analyzed the number and stability of equilibria in system (1.1) and found out that the model always predicts persistence of the

[†]the corresponding author.
Email address: yangruizhi529@163.com (R. Yang)
[1]Department of Mathematics, Northeast Forestry University, Harbin, Heilongjiang 150040, China
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parasitoid. Then, in [13], the author also considered bifurcation analysis of the following system with Holling Type II functional response. For simplicity, taking $\overline{u} = \frac{u}{K_1}$, $\overline{v} = \frac{r_2 v}{r_1 K_2}$, and $\overline{t} = r_1 t$, then (1.1) can be rewritten in the following form (still denote $\overline{u}$, $\overline{v}$, $\overline{t}$ as u, v, t respectively)

$$\begin{cases} \frac{du}{dt} = u(1 - u - \frac{bv}{a+u}), \\ \frac{dv}{dt} = v(\delta - v + \frac{cu}{a+u}), \end{cases} \tag{1.2}$$

where

$$a = \frac{1}{K_1 \xi h},\ b = \frac{K_2}{K_1 r_2 h},\ c = \frac{\gamma}{r_1 h},\ \delta = \frac{r_2}{r_1}.$$

The sufficient conditions were obtained to ensure that the equilibria are locally and globally asymptotically stable.

Time delay in population model with Holling II functional response may have significant impact on the underlying dynamics and many researchers have studied this effect [2,7,9,11,14,15,17,18]. Because of maturation time, capturing time, gestation time or other reasons, many different types of delays have been incorporated in population models. Considering the delay effect on the generalist parasitoid ,and the host and generalist parasitoid are non-homogeneous in the space. We study the following model

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = d_1 \Delta u + u - u^2 - \frac{buv}{a+u}, & x \in (0, l\pi),\ t > 0, \\ \frac{\partial v(x,t)}{\partial t} = d_2 \Delta v + v(\delta - v + \frac{cu(t-\tau)}{a+u(t-\tau)}), & x \in (0, l\pi),\ t > 0, \\ u_x(0,t) = v_x(0,t) = 0, u_x(l\pi,t) = v_x(l\pi,t) = 0,\ t > 0, & \\ u(x,\theta) = u_0(x,\theta) \geq 0, v(x,\theta) = v_0(x,\theta) \geq 0, & x \in [0, l\pi], \theta \in [-\tau, 0]. \end{cases} \tag{1.3}$$

where d_1 and d_2 are the diffusion coefficients of prey and predator respectively. The aim of this article is to study the local stability and Hopf bifurcation of the unique positive equilibrium for the system (1.3) by using τ as a parameter.

The rest of this paper is organized as follows: In Section 2, we study the local stability, Turing instability and the occurrence of Hopf bifurcation. In Section 3, we study the direction and stability of spatial Hopf bifurcation. In Section 4, we present some numerical simulations to illustrate the established results. Finally, a summarization is given in Section 5.

2. Analysis of the characteristic equations

By analyzing the associated characteristic equation at $P = (u_0, v_0)$, we investigate the stability and instability of $P = (u_0, v_0)$ for system (1.3). Denote

$$u_1(t) = u(\cdot, t), \quad u_2(t) = v(\cdot, t), \quad U = (u_1, u_2)^T,$$

$$X = C([0, l\pi], \mathbb{R}^2), \quad \text{and} \quad \mathscr{C}_\tau := C([-\tau, 0], X).$$

Linearizing system (1.3) at $P = (u_0, v_0)$, we have

$$\dot{U} = \mathbb{D}\Delta U(t) + L(U_t), \tag{2.1}$$