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Stage-structured Harvest Models*

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Abstract We formulate a stage-structured population model where the population is divided to two classes, the juveniles and the adults. Then, we include harvest in the model and assume that the harvesting is only on adults. The cases where the harvesting rate is constant, proportional to the number of adults, or of Holling-II type are studied. While the model dynamics are relatively simple when the harvesting rate is proportional, the model system with a constant or a Holling-II type harvesting rate can have multiple positive equilibria. We explore the existence of all possible equilibria and investigate their stability. We also give numerical examples to confirm our findings.

Keywords Stage structure, Population, Harvesting rate, Holling-II type, Stability.

MSC(2010) 34D20, 92D25, 92D40.

1. Introduction

Harvesting wildlife populations or fish populations is common commercially, economically or due to other reasons such as for nutrition, recreation or culture. It is a major component in many industries such as fisheries and natural resource managements. Obviously, it has nonnegligible ecological and economic impact on the nature and society. Investigations of effects of harvesting populations are important to environmental protection or extinction prevention of some species. Mathematical modeling of harvesting has a long history and plays an important role to help develop regulations and ecologically acceptable strategies in population dynamics [1, 2, 4, 5, 9]. It has also been applied to host-pathogen system [20] and predation where harvesting is on either prey or predator species or both [6, 8, 14, 15, 21, 22]. Moreover, various harvesting strategies with different harvesting rates have been assumed in many modeling works, including constant, proportional, periodic or impulsive rates [1, 7, 10–12, 18].

Those studies mentioned above are mostly based on homogeneous populations. They have gained insight to harvesting effects and possible optimal strategies. Successful mathematical analyses have also been achieved from those studies. Nevertheless, we note that in many harvest areas, harvesting is more selective in a

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lot of situations and for many species [3]. For example, people can only or tend to focus on certain sizes of fish. Hunting also has more particular preferences on certain size or age stages of animals. Thus, it is important to include age or size structure in modeling of harvesting. There are mathematical models formulated and studied with age or stage structure in the literature [16, 17, 19]. To carry on this direction, we consider a population that is divided into two stages, the juveniles and adults in this paper. We assume that harvesting is only on adults and that the harvesting rates are constantly in proportional to the number of the adults, or of Holling-II type. We explore the existence of all possible equilibria and investigate their stability. By using numerical examples, we confirm our analytic results. We finally give brief discussions as well.

2. In the absence of harvesting

First, we consider the following two-stage-structured population model in the absence of harvesting

$$\begin{aligned}\frac{dJ}{dt} &= B(J,A)A - (S(J,A) + D_1(J,A))J,\\ \frac{dA}{dt} &= S(J,A)J - D_2(J,A)A,\end{aligned}$$

where $J(t)$ and $A(t)$ denote the classes of juveniles and adults respectively, $B(J,A)$ is the per capita birth rate of adults, $S(J,A)$ is the per capita maturation rate of juveniles, and $D_1(J,A)$ and $D_2(J,A)$ are the per capita death rates of juveniles and adults respectively [13].

We assume that the birth rate of adults and the maturation rate of juveniles to adults are density-independent such that $B(J,A) := \beta$ and $S(J,A) := \alpha$. Then, we consider the ecological situation where food is unlimited for adults, but the presence of adults interferes food seeking for juveniles. Thus, the density dependence is negligible for adults but needs to be include for the juveniles death. Then, we let the death rates of juveniles and adults be given by $D_1(J,A) = d_0 + d_1J + d_2A$ and $D_2(J,A) = \mu_0$ respectively. Based on these assumptions, the model system becomes

$$\begin{aligned}\frac{dJ}{dt} &= \beta A - \alpha J - (d_0 + d_1J + d_2A)J,\\ \frac{dA}{dt} &= \alpha J - \mu_0 A.\end{aligned} \tag{2.1}$$

Let $N = J + A$. Then,

$$\frac{dN}{dt} = \beta A - (d_0J + \mu_0A) - (d_1J + d_2A)J \le \beta N - \bar{d}_0N - \bar{d}_1N^2,$$

where we let

$$\bar{d}_0 := \min\{d_0, \mu_0\} \quad \text{and} \quad \bar{d}_1 := \min\{d_1, d2\}.$$

We assume $\beta > \bar{d}_0$ and define set Ω as

$$\Omega := \left\{(J,A); 0 \le J + A \le \frac{\beta - \bar{d}_0}{\bar{d}_1}\right\}. \tag{2.2}$$