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Application of Almost Increasing Sequence for Absolute Riesz $|\overline{N}, p_n^{\alpha,\beta}; \delta|_k$ Summable Factor*

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Abstract In this paper, we generate an extended result by Bor and Seyhan concerning absolute Riesz summability factors. Further, we develop some well-known results from our main result.

Keywords Absolute summability, Quasi-f-power increasing sequence, Infinite series, Riesz summability.

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1. Introduction

Let $\sum a_n$ be an infinite series, $\{s_n\}= \sum\limits_{k=0}^{n} a_k$ be the sequence of its partial sums and n^{th} mean of the sequence $\{s_n\}$ is given by u_n, s.t.,

$$u_n = \sum_{k=0}^{\infty} u_{nk} s_k. \tag{1.1}$$

$$\tag{1.2}$$

Definition 1: An infinite series $\sum a_n$ is absolute summable, if

$$\lim_{n\to\infty} u_n = s$$

and

$$\sum_{n=1}^{\infty} |u_n - u_{n-1}|<\infty, \tag{1.3}$$

Definition 2: Let $\{p_n\}$ be a sequence with $p_0 > 0$ and $p_n \geqslant 0$ for $n > 0$

$$P_n = \sum_{v=0}^{n} p_v \to \infty. \tag{1.4}$$

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For $\alpha > -1$, $0 < \beta \leqslant 1$, $\alpha + \beta > 0$, define:

$$\in_0^{\alpha+\beta} = 1,\ \in_n^{\alpha+\beta} = \frac{(\alpha+\beta+1)(\alpha+\beta+2)....(\alpha+\beta+n)}{n!},\ (n = 1, 2, 3, ...) \tag{1.5}$$

$$p_n^{\alpha,\beta} = \sum_{v=0}^{n} \in_{n-v}^{\alpha+\beta-1} p_v, \tag{1.6}$$

$$P_n^{\alpha,\beta} = \sum_{v=0}^{n} p_n^{\alpha,\beta} \to \infty,\ n \to \infty \tag{1.7}$$

and

$$P_{-n}^{\alpha,\beta} = p_{-n}^{\alpha,\ \beta} = 0,\ n \geqslant 1.$$

Then, the sequence-to-sequence transformation t_n defines the $(\overline{N}, p_n^{\alpha,\beta})$ mean of series $\sum a_n$ and is given by:

$$t_n = \frac{1}{P_n^{\alpha,\beta}} \sum_{k=0}^{n} p_k^{\alpha,\beta} s_k,\ P_n^{\alpha,\beta} \neq 0,\ n \in N \tag{1.8}$$

and $\lim_{n\to\infty} t_n = s$, and the series is called $(\overline{N}, p_n^{\alpha,\beta})$, formed by sequence of coefficients $\{p_n^{\alpha,\beta}\}$.
Further, if sequences $\{t_n\}$ is of bounded variation with index $k \geqslant 1$ i.e.

$$\sum_{n=1}^{\infty} \left(\frac{P_n^{\alpha,\beta}}{p_n^{\alpha,\beta}}\right)^{k-1} |\Delta t_{n-1}|^k < \infty, \tag{1.9}$$

then the series $\sum a_n$ is said to be absolutely $(R, p_n^{\alpha,\beta})_k$ summable with index k or $|\overline{N}, p_n^{\alpha,\beta}|_k$ summable to s.
Definition 3: The series is said to be $|\overline{N}, p_n^{\alpha,\beta}; \delta|_k$ summable, if

$$\sum_{n=1}^{\infty} \left(\frac{P_n^{\alpha,\beta}}{p_n^{\alpha,\beta}}\right)^{\delta k+k-1} |\Delta t_{n-1}|^k < \infty, \tag{1.10}$$

with $k \geqslant 1$, $\delta \geqslant 0$ and

$$\Delta t_n = -\frac{p_n^{\alpha,\beta}}{P_n^{\alpha,\beta} P_{n-1}^{\alpha,\beta}} \sum_{v=1}^{n} P_{v-1}^{\alpha,\beta} a_v,\quad n \geqslant 1. \tag{1.11}$$

Bor [1–3] generalised the result associated with Riesz summability factors. Bor and Özarslan [4,5] established theorems using $|\overline{N}, p_n; \delta|$ summability factors. Özarslan [11, 12] used the definition of almost increasing sequence for absolute summability. Mishra et. al. [9, 10] gave useful result on approximation. Also, Mishra et. al. [7, 8] provided new results related to matrix summability and improper integrals. In [13], Sonker and Munjal established new theorem on absolute summability for Triangle matrices. Yildiz [14, 15] determined theorems on generalized absolute matrix summability factors.