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A Switching Law to Stabilize an Unstable Switched Linear System*

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Abstract Stabilization of switched systems fully composed of unstable modes is of theoretical and practical significance. In this paper, we obtain some sufficient algebraic conditions for stabilizing switched linear systems with all unstable subsystems based on the theory of spherical covering and crystal point groups. Under the proposed algebraic conditions switching laws are easy to be designed to stabilize the switched systems. Some simple examples are provided to illustrate our results.

Keywords Switched system, Unstable mode, Stabilization, Switching law, Spherical covering.

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1. Introduction

A lot of natural and artificial processes encompass several modes of operation with a different dynamical behavior in each mode. To deal with such processes, theory of hybrid dynamical systems has been developed in recent years. A typical class of hybrid systems are the so-called switched systems. A switched system is composed of a family of subsystems and rules that regulate the switching among them. Stability issue for switched systems is of great significance, which has been extensively studied [2, 4, 11–13, 15, 16, 29]. One of the early results of hybrid system stability for linear switched systems was developed by Peleties [15]. In [17], necessary and sufficient conditions are proposed for a given set of controllers to quadratically stabilize a plant and for robust stabilizability with a quadratic storage function. Also, the stability condition can be expressed from LyapunovMetzler inequalities [7]. For more details about switching in system and control, see [10, 18, 20].

Stabilization of switched systems fully consisting of unstable modes is one of the most challenging problems in the field of switched systems [5, 25, 27]. Multiple Lyapunov or Lyapunov-likes functions [11, 12] may be concatenated together to produce a nontraditional Lyapunov function and stabilize unstable switched system.

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Decarlo et al. [5] proposed conditions for the existence of switched controllers in stabilizing unstable switched systems by means of Lyapunov-like functions. The corresponding Lyapunov-like function values at every switching time form a monotonically decreasing sequence. For a switched system with all unstable subsystems, how to design appropriate switching laws to stabilize the switched system is of great interest. Except with multiple Lyapunov functions (MLF) theory, the switching signal can be designed to stabilize the unstable system with mode-dependent average dwell time (MDADT) property [30].

However, for a system, its corresponding Lyapunov function may be hard to construct even though it does exist. Therefore, studying simple sufficient conditions to ensure that a switched system with all unstable modes is stabilizable is of very importance. Motivated by [1, 10], we investigate a new easy-to-use sufficient condition for stabilization of n-dimensional switched systems with all unstable modes and obtain some novel results in two-dimensional and three-dimensional switched systems by virtue of the theory of spherical covering and crystal point groups [3, 6].

This paper is organized as follows: In Section 2, we set forth a sufficient condition for stabilizing switched linear systems with all unstable subsystems. Section 3 presents a sufficient condition for stabilization of two-dimensional switched systems with all unstable modes. Two examples are provided to illustrate the ideas in this section. Section 4 presents some results about stabilization of three-dimensional switched systems with three, four and five unstable modes.

2. A new sufficient condition for stabilization

Before starting our discussions, we give some notations. Let $\mathbf{M}_{n\times n}$ denote the set of all $n \times n$ real matrices and $\mathbf{M}^u_{n\times n}$ denote the matrices in $\mathbf{M}_{n\times n}$ with at least one of its eigenvalues having positive real parts. We denote the inner product of two vectors by $\langle\cdot,\cdot\rangle$, the transpose of $\mathbf{A}$ by $\mathbf{A}^T$ and the Euclidean norm on $\mathbb{R}^n$ by $|\cdot|$. Denote the unit circle in $\mathbb{R}^2$ by S^1 and the unit sphere in $\mathbb{R}^3$ by S^2.

In this paper, we focus on the following linear switched system

$$\dot{\mathbf{x}} = \mathbf{A}_{\sigma(\mathbf{x})}\mathbf{x} \tag{2.1}$$

where $\mathbf{x} \in \mathbb{R}^n$ and $\sigma(\mathbf{x}) : \mathbb{R}^n \to \mathcal{M} = \{1, 2, \ldots, m\}$ denotes a switching function where m is the number of modes and $\mathbf{A}_i \in \mathbf{M}^u_{n\times n}, i \in \mathcal{M}$. For system (2.1) with state-dependent switching, the state space $\mathbb{R}^n$ is partitioned into a finite number of operating regions by means of a family of switching surface.

Based on Lyapunov stability theory [17, 26], we have the definition of stabilizability as follows.

Definition 2.1. The origin of a switched system (2.1) is said to be switching stabilizable if there exists a switching law $\sigma : \mathbb{R}^n \longmapsto \{1, 2, \ldots, m\}$ under which the origin is asymptotically stable.

According to the results in [17], we have the following definition.

Definition 2.2. The collection of real symmetric matrices $\mathbf{Z}_1, \mathbf{Z}_2, \ldots, \mathbf{Z}_k$ is said to be complete if for any $\mathbf{x}_0 \in \mathbb{R}^n$ there exists $i \in \{1, 2, \ldots, k\}$ such that $\mathbf{x}_0^T\mathbf{Z}_i\mathbf{x}_0 \le 0$. Furthermore, the collection $\mathbf{Z}_1, \mathbf{Z}_2, \ldots, \mathbf{Z}_k$ is said to be strictly complete if for any $\mathbf{x}_0 \in \mathbb{R}^n/\{\mathbf{0}\}$ there exists $i \in \{1, 2, \ldots, k\}$ such that $\mathbf{x}_0^T\mathbf{Z}_i\mathbf{x}_0 < 0$.