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WEAK AND STRONG CONVERGENCE THEOREMS FOR SPLIT GENERALIZED MIXED EQUILIBRIUM PROBLEM*†

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Abstract

The purpose of this paper is to introduce a split generalized mixed equilibrium problem (SGMEP) and consider some iterative sequences to find a solution of the generalized mixed equilibrium problem such that its image under a given bounded linear operator is a solution of another generalized mixed equilibrium problem. We obtain some weak and strong convergence theorems.

Keywords split generalized mixed equilibrium problem; weak convergence; strong convergence; fixed point

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1 Introduction and Preliminaries

Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$ and C be a nonempty closed convex subset of H. Let f be a bi-function from $C\times C$ to R and $\varphi : C\to R$ be a function, where R is the set of real numbers. Let $B : C\to H$ be a nonlinear mapping. Then we consider the following generalized mixed equilibrium problem: There exists an $x\in C$, such that

$$f(x,y)+\varphi(y)-\varphi(x)+\langle Bx, y-x\rangle \ge 0, \quad \text{for any } y\in C. \tag{1.1}$$

The set of solutions of (1.1) is denoted by $GMEP(f,\varphi,B)$.

If $B=0$, problem (1.1) becomes the following mixed equilibrium problem: There exists an $x\in C$, such that

$$f(x,y)+\varphi(y)-\varphi(x)\ge 0, \quad \text{for any } y\in C. \tag{1.2}$$

The set of solutions of (1.2) is denoted by $MEP(f,\varphi)$.

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If $\varphi = 0$, problem (1.1) reduces to the following generalized equilibrium problem: There exists an $x \in C$, such that

$$f(x,y) + \langle Bx, y - x\rangle \geq 0, \quad \text{for any } y \in C. \tag{1.3}$$

The set of solutions of (1.3) is denoted by $GEP(f, B)$.

If $\varphi = 0$ and $B = 0$, problem (1.1) becomes the following equilibrium problem: There exists an $x \in C$, such that

$$f(x,y) \geq 0, \quad \text{for any } y \in C. \tag{1.4}$$

The set of solutions of (1.4) is denoted by $EP(f)$.

Equilibrium problem is very general in the sense that it includes, as special cases, optimization problems, variational inequalities, mini or max problems, Nash equilibrium problem in noncooperative games and others; see for instance [1-20].

In 2012, Zhenhua He [12] proposed a new equilibrium problem which is called split equilibrium problem (SEP). Let E_1 and E_2 be two real Banach spaces, C be a closed convex subset of E_1, K be a closed convex subset of E_2, $A : E_1 \to E_2$ be a bounded linear operator, f be a bi-function from $C \times C$ into R and g be a bi-function from $K \times K$ into R. The SEP is to find an element $x^* \in C$, such that

$$f(x^*, y) \geq 0, \quad \text{for any } y \in C,$$

and such that $u := Ax^* \in K$ satisfying

$$g(u, v) \geq 0, \quad \text{for any } v \in K.$$

Inspired and motivated by the above works, we propose a split generalized mixed equilibrium problem (SGMEP). Let E_1 and E_2 be two real Banach spaces, E_1^* and E_2^* denote the dual of E_1 and E_2, respectively, C be a closed convex subset of E_1, K be a closed convex subset of E_2, $A : E_1 \to E_2$ be a bounded linear operator, f be a bi-function from $C \times C$ into R, g be a bi-function from $K \times K$ into R, $B : C \to E_1^*$ and $S : K \to E_2^*$ be two mappings, $\varphi : C \to R$ and $\psi : K \to R$ be two functions. The SGMEP is to find an element $p \in C$ such that

$$f(p, y) + \varphi(y) - \varphi(p) + \langle Bp, y - p\rangle \geq 0, \quad \text{for any } y \in C, \tag{1.5}$$

and that $u := Ap \in K$ satisfies

$$g(u, v) + \psi(v) - \psi(u) + \langle Su, v - u\rangle \geq 0, \quad \text{for any } v \in K. \tag{1.6}$$

For convenience, we denote the solution set of the SGMEP by Ω, that is, $\Omega = \{x \in GMEP(f, \varphi, B) : Ax \in GMEP(g, \psi, S)\}$.

Now, we give two examples to show $\Omega \neq \emptyset$.