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# NASH EQUILIBRIUM OF TWO GAME MODELS ON THE RANDOM SOCIAL NETWORK*†

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**Abstract**

It is known that somebody's behavior (decision) in a stochastic social network may be influenced by that of his (or her) friends. In this paper, we consider two stochastic social network game models (a) and (b) which can be defined respectively by two different utility functions. Some sufficient conditions for the existence of Nash equilibrium (NE) of the two network game models are obtained by analyzing the different effort relation between a player and his (or her) neighbors.

**Keywords** degree distribution; utility function; Nash equilibrium

**2000 Mathematics Subject Classification** 91D30

## 1 Introduction

Network game is one of the fundamental and important research problems in social networks, since it plays a key role in studying the evolution and the equilibrium of random systems on the networks. In the past twenty years, specially in the recent years, various games, especially the opinion dynamics, on different kinds of networks have been addressed and studied by many researchers.

Jackson and Wolinsky ([15], 1996) and Jackson and Watts ([14], 2002) presented and studied the stability and efficiency of random social networks. They also examined the dynamic formation and stochastic evolution of the networks. Newman ([20], 2010) presented a general dynamic systems of continuous variables evolving in continuous time on networks. Galeotti et al. ([9], 2009; [10], 2011) analyzed a strategy of the influence between the individual and his neighbors, and presented an approach to obtain the existence of Nash equilibrium of a random network. Gharehshiran et al. ([12], 2014) investigated the problems of reinforcement learning

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and nonparametric detection of game equilibrium of social networks. Rebille and Richefort ([21], 2014) proved the existence and uniqueness of the equilibrium for network games with additive preferences. Cimini et al ([6], 2015) showed by numerical simulations that when the players imitate each other, where an individual's welfare depends on both her and her partners' actions, evolution does not reach Nash equilibrium. Kordonis and Papavassilopoulos ([16], 2017) introduced a probabilistic approximate Nash equilibrium notion to study static and dynamic games on large networks of interacting agents.

Opinion formation in social networks is an important area of research that has attracted a lot of attention in a wide range of disciplines, such as psychology, economics, political science, mathematics and computer engineering. Acemoglu and Ozdaglar ([1], 2011) provided an overview of research work prior to 2010 on belief and opinion dynamics in social networks. Ghaderi and Srikant ([11], 2014) studied whether an equilibrium can emerge and how such equilibrium possibly depends on the network structure, initial opinions of the agents, and the location of stubborn agents. Belhaj et al. ([4], 2014) analyzed linear network games under strategic complementarities with an upper bound on actions and showed that there is always a unique equilibrium. Bindel et al. ([5], 2015) considered an opinion game on social network in which each person (node) holds an opinion that will be influenced by the opinions of his (her) neighbors, and showed that the opinion game has a unique Nash equilibruim to which the repeated averaging process converges. Etesami and Basar ([8], 2015) considered the synchronous Hegselmann-Krause model for opinion dynamics in finite dimensions and provided a polynomial upper bound for the termination time of the synchronous dynamics. Liakos and Papakonstantinopoulu ([18], 2016) analyzed spreading of opinions in social networks and showed experimentally that the repeated averaging process results to Nash equilibrium. Grandi et al. ([13], 2017) extended a simple model of opinion diffusion on networks with a strategic component and showed how apparently simple problems in the strategic opinion diffusion require a complex logical machinery to be properly formalized and handled. Li et al. ([17], 2017) considered a new opinion formation model of heterogeneous agents and showed by numerical simulations that opinion guidance is most likely to separate the public into different groups rather than converge to the guide's opinion. Proskurnikov and Tempo ([19], 2017) provided a survey of several dynamic models for opinion formation with recent results of the convergence and stability properties on multi-agent systems. Albi et al. ([2], 2017) surveyed some recent developments on the mathematical modeling of opinion dynamics.

All the above research work mainly focused on the study of non-random dynamic system on social network except the work of Jackson and Watts and Galeotti et