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DYNAMIC BEHAVIOURS OF AN AUTONOMOUS STAGE-STRUCTURED COMPETITIVE SYSTEMS WITH TOXIC EFFECT*†

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Abstract

An autonomous stage-structured competitive systems with toxic effect is investigated in this paper. Sufficient conditions which guarantee the global attractivity of the system and the extinction of the partial species are obtained, respectively. Our results supplement and compliment one of the main results of Liu and Li [Global stability analysis of a nonautonomous stage-structured competitive system with toxic effect and double maturation delays, Abstract and Applied Analysis, Volume 2014, Article ID 689573, 15 pages].

Keywords global attractivity; extinction; delay; toxic substance

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1 Introduction

Throughout this paper, we set:

$$f^M = \max_{t\in[0,\omega]} |f(t)|, \quad f^L = \min_{t\in[0,\omega]} |f(t)|,$$

where $f(t)$ is a ω-periodic continuous function.

It is well known that the effect of toxins on ecological systems is an important issue from mathematical and experimental points of view [1, 2]. In [3], Maynard Smith incorporated the effects of toxic substances in a two-species Lotka-Volterra competitive system by considering that each species produces a substance toxic to the other only when the other is present. The model takes the following form

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$$\begin{aligned} \dot{x}_1(t) &= x_1(t)[K_1 - a_1x_1(t) - b_1x_2(t) - c_1x_1(t)x_2(t)], \\ \dot{x}_2(t) &= x_2(t)[K_2 - a_2x_2(t) - b_2x_1(t) - c_2x_1(t)x_2(t)], \end{aligned} \tag{1.1}$$

where $x_1(t)$ and $x_2(t)$ represent the densities of two competing species at time t, respectively. K_1 and K_2 denote the birth rates of the first and second species, respectively. a_1 and a_2 are the rates of intraspecific competition term for the first and second species, respectively. b_1 and b_2 stand for the rates of interspecific competitions, respectively. c_1 and c_2 represent the toxic inhibition rates for the first species by the second species and vice versa.

However, the nonautonomous case is more realistic, according to system (1.1), Li and Chen [4] considered the following nonautonomous system of differential equations

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)[b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - d_1(t)x_1(t)x_2(t)], \\ \dot{x}_2(t) &= x_2(t)[b_2(t) - a_{21}(t)x_2(t) - a_{22}(t)x_1(t) - d_2(t)x_1(t)x_2(t)]. \end{aligned} \tag{1.2}$$

They showed that under some suitable conditions, one species will be driven to extinction while the other species stabilizes at a certain solution of a logistic equation. For more papers in this direction, one could refer to [5,7], [24,25] and the references cited therein.

Stage-structured models have been analyzed in many papers (see [8,12-20,23]). Recently, Li and Chen [8] proposed the following periodic competitive stage-structured Lotka-Volterra model with the effects of toxic substances

$$\begin{aligned} \dot{x}_1(t) &= b_1(t-\tau_1)\exp\Big(-\int_{t-\tau_1}^{t} r_1(s)\mathrm{d}s\Big)x_1(t-\tau_1) - a_{11}(t)x_1^2(t) \\ &\quad -a_{12}(t)x_1(t)x_2(t) - d_1(t)x_1^2(t)x_2(t), \\ \dot{y}_1(t) &= b_1(t)x_1(t) - r_1(t)y_1(t) - b_1(t-\tau_1)\exp\Big(-\int_{t-\tau_1}^{t} r_1(s)\mathrm{d}s\Big)x_1(t-\tau_1), \\ \dot{x}_2(t) &= b_2(t-\tau_2)\exp\Big(-\int_{t-\tau_2}^{t} r_2(s)\mathrm{d}s\Big)x_2(t-\tau_2) - a_{21}(t)x_1(t)x_2(t) \\ &\quad -a_{22}(t)x_2^2(t) - d_2(t)x_1(t)x_2^2(t), \\ \dot{y}_2(t) &= b_2(t)x_2(t) - r_2(t)y_2(t) - b_2(t-\tau_2)\exp\Big(-\int_{t-\tau_2}^{t} r_2(s)\mathrm{d}s\Big)x_2(t-\tau_2), \end{aligned} \tag{1.3}$$

where $x_i(t)$ and $y_i(t)$ $(i = 1, 2)$ represent the densities of mature and immature species at time $t > 0$, respectively. $b_i(t)$, $a_{ij}(t)$, $r_i(t)$, $d_i(t)$ $(i, j = 1, 2)$ are all nonnegative continuous and ω-periodic functions. They obtained a set of sufficient conditions which ensure the extinction of the second species and the global attractivity of the first species.