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PERMANENCE OF PERIODIC BEDDINGTON-DEANGELIS PREDATOR-PREY SYSTEM IN A TWO-PATCH ENVIRONMENT WITH DELAY*†

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Abstract

In this paper, we study a two-species periodic Beddington-DeAngelis predator-prey model with delay in a two-patch environment, in which the prey species can disperse between two patches, but the predator species cannot disperse. On the basis of the comparison theorem of differential equations, we establish sufficient conditions for the permanence and extinction of the system.

Keywords predator-prey system; Beddington-DeAngelis functional response; permanence; time delay

2000 Mathematics Subject Classification 34D23

1 Introduction

The purpose of this paper is to consider the following Beddington-DeAngelis predator-prey system with time delay in a two-patch environment

$$\begin{cases} \dot{x}_1(t) = x_1(t)\left(a_1(t) - b_1(t)x_1(t) - \dfrac{\beta y(t)}{1+\gamma_1(t)x_1(t)+\gamma_2(t)y(t)}\right) + d_1(t)(x_2(t)-x_1(t)), \\ \dot{x}_2(t) = x_2(t)(a_2(t) - b_2(t)x_2(t)) + d_2(t)(x_1(t) - x_2(t)), \\ \dot{y}(t) = y(t)\left(-e(t) + \dfrac{\beta x_1(t-\tau(t))}{1+\gamma_1(t)x_1(t-\tau(t))+\gamma_2(t)y(t-\tau(t))}\right), \end{cases} \tag{1.1}$$

where $x_i(t)$ $(i = 1, 2)$ and $y(t)$ are the population densities in i-th patch and predator species in 1-th patch, respectively. The positive functions $a_i(t)$ and $b_i(t)$ $(i = 1, 2)$ represent the intrinsic growth rate and the density-dependent coefficient of the prey species x in i-th patch, respectively. The function $d_i(t)$ $(i=1, 2)$ denotes the dispersal

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rate of the prey in the i-th patch. The function $\frac{\beta x_1(t)y(t)}{1+\gamma_1(t)x_1(t)+\gamma_2(t)y(t)}$, where $\gamma_1(t)$ and $\gamma_2(t)$ are positive functions, represents the Beddington-DeAngelis infection rate. The function $\tau(t)$ is the time delay, and $e(t)$ is the death rate of the predator.

The Beddington-DeAngelis functional response in (1.1) was introduced by Beddington [1] and DeAngelis [4]. Many authors has studied this functional response. For example, Fan and Kuang [10] studied a nonautonomous predator-prey system with Beddington-DeAngelis functional response. They established two sufficient criteria for the existence of a positive solution by using Brouwer fixed point theorem and continuation theorem in coincidence degree, respectively. See also [16,17] for related works on predator-prey systems with Beddington-DeAngelis functional response.

Besides the functional responses mentioned above, there are many ratio-dependent functional responses, such as Holling type and Gause-type ratio-dependent functional responses. Especially, Ding et al. [8] considered a two-species periodic Gause-type ratio-dependent predator-prey system with time delay in a two-patch environment. By using the comparison theorem of differential equations, [8] established sufficient conditions for the permanence of the system. Related works on predator-prey system in a two-patch environment can also be found in [6,19,25,27]. For more works on Gause-type ratio-dependent predator-prey systems, we refer to [5,7,14].

We also note that the ratio-dependent predator-prey model has been studied by many authors [2,4,9,11-13]. Especially, Chen and Shi [2] discussed a special case of ratio-dependent predator-prey system, and obtained sufficient conditions for the permanence and existence of positive periodic solution. Fan and Li [11] considered the global asymptotic stability of a ratio-dependent predator-prey system with diffusion. In [12], Fan and Li studied the permanence of delayed ratio-dependent predator-prey models with monotonic functional responses. We can also find the similar results in [23,24].

Motivated by the previous works, in this paper by incorporating the ratio-dependent Beddington-DeAngelis functional response into the system studied in [8], we consider the Beddington-DeAngelis predator-prey system (1.1) with time delay in a two-patch environment. By the well-known result for periodic Logistic equation and the comparison theorem, we establish sufficient conditions for the permanence and extinction of system (1.1). We note that the method to prove the permanence of the prey in this paper is different from [8].

For convenience, we assume that $a_i(t)$, $b_i(t)$, $d_i(t)$, $\gamma_i(t)$ $(i = 1, 2)$, $e(t)$ and $\tau(t)$ are positive continuous periodic functions with a common period $\omega > 0$ throughout the paper.

By the theory of functional differential equations, system (1.1) with initial con-