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CONTROLLABILITY OF QUASI-LINEAR IMPULSIVE FUNCTIONAL BOUNDARY VALUE PROBLEMS*†

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Abstract

By employing the Schauder fixed-point theorem, we establish new sufficient conditions for the controllability of impulsive functional boundary value problems.

Keywords controllability; boundary value problems (BVPs); fixed points
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1 Introduction

In practical control systems, impulses exist widely involving almost all fields such as medicine, biology, economics, electronics and etc. And hence this kind of systems has attracted considerable interest during the past decades. In general, as reported in Lakshmikanthan, Bainov and Simeonov [1], impulsive systems combine continuous evolution with instantaneous state jumps or resets. These systems provide a natural framework for mathematical modeling of many real world evolutionary processes where the states undergo abrupt changes at certain instants or at variable instants.

The concept of controllability plays an important role in control theory and engineering, and the problem of controllability of boundary value problems represented by functional differential equations has been extensively studied (see Han and Park [2], Akhmetov, Perestyuk and Tleubergenova [3], Akhmetov and Zafer [4] and Balachandran, Dauer [5]). In Lando [6], a method was suggested for solving problems of control over linear systems based on the normal solvability of boundary value problems. Akhmetov and Zafer [4] developed the above ideas for impulsive system

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$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}x(t) = A(t)x(t) + C(t)u + f(t) + \mu g(t,x,u,\mu), \quad t \neq \theta_i, \\ \Delta x|_{t=\theta_i} = B_i x + D_i v_i + J_i + \mu W_i(x, v_i, \mu), \\ x(\alpha) = a, \quad x(\beta) = b, \end{cases} \tag{1.1}$$

and obtained the controllability of system (1.1) by contraction mapping principle.

For fixed real numbers α and β with $\alpha < \beta$ and fixed positive integers r and p, denote by $L_2^r[\alpha,\beta]$ the set of all square integrable functions $\eta : [\alpha,\beta] \to \mathbb{R}^r$ and by $D^r[1,p]$ the set of all finite sequences $\{\xi_i\}$, $\xi_i \in \mathbb{R}^r$, $i = 1, \cdots, p$. We define a space $\Pi_p^r = L_2^r \times D^r$ whose elements are denoted by $\{\eta,\xi\}$ and let

$$\langle\{\eta,\xi\},\{\omega,v\}\rangle = \int_\alpha^\beta (\eta,\omega)\mathrm{d}t + \sum_{i=1}^p (\xi_i, v_i)$$

be an inner product in Π_p^r, where $(\cdot,\cdot)$ is the Euclidean scalar product in $\mathbb{R}^r$.

Set

$$PC(I,\mathbb{R}^n) = \{x : I \to \mathbb{R}^n,\ x(t) \text{ is continuous everywhere expect for a finite number of points } \widetilde{t} \text{ at which } x(\widetilde{t}^-), x(\widetilde{t}^+) \text{ exist and } x(\widetilde{t}^-) = x(\widetilde{t})\}.$$

If $x \in PC([-\tau,T],\mathbb{R}^n)$, then for each $t \in [0,T]$, we define $x_t \in PC([-\tau,0],\mathbb{R}^n)$ by $x_t(\theta) = x(t+\theta)$ for $-\tau \le \theta \le 0$.

In this paper, we investigate the controllability of the following impulsive functional boundary value problems

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}[x(t) - h(t,x_t)] = A(t)x(t) + C(t)u + f(t,x_t) + \mu g(t,x_t,u), \quad t \neq \theta_i, \\ \Delta x(t) = B_i x(t) + D_i v_i + J_i + \mu W_i(x, v_i, \mu), \quad t = \theta_i, \\ x(t) = \phi(t), \quad t \in [-\tau, 0], \\ x(T) = b, \end{cases} \tag{1.2}$$

where μ is a small positive parameter, both τ and T are positive constants, $x \in \mathbb{R}^n$, the symbol $\Delta(\theta)$ means $x(\theta^+) - x(\theta^-)$ with $x(\theta^+) = \lim\limits_{t\to\theta+0} x(t)$ and $x(\theta^-) = \lim\limits_{t\to\theta-0} x(t)$, $\phi \in C_\tau = PC([-\tau,0],\mathbb{R}^n)$, $A(t)$ and $C(t)$ are the known matrices of the sizes $(n\times n)$ and $(n\times m)$, respectively, the elements of which belong to $L_2^1[0,T]$, both B_i and D_i are constant matrices of size $(n\times n)$ with $\det(I_i + B_i) \neq 0$ $(i = 1,\cdots,p)$, $\{\theta_i\}$ $(i = 1,\cdots,p)$ is a strictly increasing sequence of real numbers in $(0,T)$ with $0 < \theta_1 < \theta_2 < \cdots < \theta_p < \theta_{p+1} = T$, $b, J_i, v_i \in \mathbb{R}^n$ are all constant vectors, $J = \{J_i\}$, $v = \{v_i\}$ $(i = 1,\cdots,p)$. $f : [0,T] \times C_\tau \to \mathbb{R}^n$, $g : [0,T] \times C_\tau \times \mathbb{R}^m \to \mathbb{R}^n$, $h : [0,T] \times C_\tau \to \mathbb{R}^n$, $W_i : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ $(i = 1,\cdots,p)$, $\{f,J\} \in \Pi_p^n[0,T]$. Set $G_0 = [0,\theta_1]$, $G_k = (\theta_k, \theta_{k+1}]$ $(k = 1,\cdots,p)$.

Definition 1.1 The boundary value problem (1.2) is said to be controllable, if for any $\phi \in C_\tau$, $b \in \mathbb{R}^n$, there exists a $\{u,v\} \in \Pi_p^m$ for which the boundary value problem