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PERSISTENCE AND EXTINCTION OF A STOCHASTIC SIS EPIDEMIC MODEL WITH DOUBLE EPIDEMIC HYPOTHESIS*

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Abstract

In this paper, we aim at dynamical behaviors of a stochastic SIS epidemic model with double epidemic hypothesis. Sufficient conditions for the extinction and persistence in mean are derived via constructing suitable functions. We obtain a threshold of stochastic SIS epidemic model, which determines how the diseases spread when the white noises are small. Numerical simulations are used to illustrate the efficiency of the main results of this article.

Keywords double epidemic hypothesis; Brownian motion; extinction; persistence

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1 Introduction

Epidemiology is the science of studying the spread of infectious diseases, which is to investigate and to trace the dynamics and stabilities of infectious diseases. The modified models and recent contributions always assume that the population is separated by three subclasses: the susceptible, infective and the recovered, denoting them as S, I and R respectively.

The classical SIS model turns into SIR model or SIRS model when the recovered individuals are taken into account. Related research and modified models can be found in [1-5]. When the exposed individuals are considered into population level and participate into the spread process of disease, the classical SIS model becomes a new version, often mentioned as SEIR model or SEIRS model if the recovered individuals return into the susceptible again, for instance, see the recent literatures [6-9].

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Meng *et al.* [10] discussed an SIS epidemic model with double epidemic hypothesis of the following form:

$$\begin{cases} \dot{S}(t) = A - \mu S(t) - \dfrac{\beta_1 S(t) I_1(t)}{a_1 + I_1(t)} - \dfrac{\beta_2 S(t) I_2(t)}{a_2 + I_2(t)} + r_1 I_1(t) + r_2 I_2(t), \\ \dot{I}_1(t) = \dfrac{\beta_1 S(t) I_1(t)}{a_1 + I_1(t)} - (\mu + \alpha_1 + r_1) I_1(t), \\ \dot{I}_2(t) = \dfrac{\beta_2 S(t) I_2(t)}{a_2 + I_2(t)} - (\mu + \alpha_2 + r_2) I_2(t), \end{cases} \tag{1.1}$$

where A is the total input susceptible population size, β_1 and β_2 are the contact rates, μ is the natural mortality, α_1 and α_2 are the rates of disease-related death, r_1 and r_2 are the treatment cure rates of two diseases, respectively. Functions $\frac{\beta_1 S(t) I_1(t)}{a_1 + I_1(t)}$ and $\frac{\beta_2 S(t) I_2(t)}{a_2 + I_2(t)}$ respectively represent saturated incidence rates for two epidemic diseases. Model (1.1) admits the following equilibria:

$$E_0 : \Big(\frac{A}{\mu}, 0, 0\Big),$$

$$E_1 : (S_1^*, I_1^*, 0) \text{ with } S_1^* = \frac{(\mu+\alpha_1+r_1)(\alpha_1+I_1^*)}{\beta_1}, \quad I_1^* = \frac{\beta_1 A - a_1\mu(\mu+\alpha_1+r_1)}{\mu(\mu+\alpha_1+r_1)+\beta_1(\mu+a_1)},$$

$$E_2 : (S_2^*, 0, I_2^*) \text{ with } S_2^* = \frac{(\mu+\alpha_2+r_2)(\alpha_2+I_2^*)}{\beta_2}, \quad I_2^* = \frac{\beta_2 A - a_2\mu(\mu+\alpha_2+r_2)}{\mu(\mu+\alpha_2+r_2)+\beta_2(\mu+a_2)},$$

$$E^* : (S^*, I_1^*, I_2^*) \text{ with } S^* = \frac{A + a_1(\mu+\alpha_1) + a_2(\mu+\alpha_2)}{\mu + \frac{\beta_1(\mu+\alpha_1)}{\mu+\alpha_1+r_1} + \frac{\beta_2(\mu+\alpha_2)}{\mu+\alpha_2+r_2}},$$

$$I_1^* = \frac{\beta_1 A - a_1\mu(\mu+\alpha_1+r_1) + (\mu+\alpha_2)(\beta_1 a_2 - \beta_2 a_1)}{\mu(\mu+\alpha_1+r_1) + \beta_1(\mu+\alpha_1) + \beta_2(\mu+\alpha_2)},$$

$$I_2^* = \frac{\beta_2 A - a_2\mu(\mu+\alpha_2+r_2) + (\mu+\alpha_1)(\beta_2 a_1 - \beta_1 a_2)}{\mu(\mu+\alpha_2+r_2) + \beta_2(\mu+\alpha_2) + \beta_1(\mu+\alpha_1)}.$$

Let

$$\mathcal{R}_1 = \frac{\beta_1 A}{a_1\mu(\mu+\alpha_1+r_1)}, \quad \mathcal{R}_2 = \frac{\beta_2 A}{a_2\mu(\mu+\alpha_2+r_2)}$$

be the thresholds of model (1.1). Meng *et al.* [10] derived that: (i) If $\mathcal{R}_1 < 1$ and $\mathcal{R}_2 < 1$, then two diseases go extinct and model (1.1) has a unique stable diseases-extinction equilibrium point E_0. (ii) If $\mathcal{R}_1 > 1$ and $\mathcal{R}_2 < 1$, then the disease I_2 is extinct and model (1.1) has a unique stable equilibrium E_1. (iii) If $\mathcal{R}_1 < 1$ and $\mathcal{R}_2 > 1$, then the disease I_1 is extinct and model (1.1) has a unique stable equilibrium E_2. (iv) When (1.1) has a positive equilibrium E^*, if $\mathcal{R}_1 > 1$ and $\mathcal{R}_2 > 1$, then E^* is a unique stable equilibrium, which implies two diseases of model (1.1) are persistent.

The main aim of this article is to investigate how the dynamics behaviors when the environmental noise is considered in deterministic model (1.1). Let $B_i(t)$ $(i =$