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STABILITY ANALYSIS OF A LOTKA-VOLTERRA COMMENSAL SYMBIOSIS MODEL INVOLVING ALLEE EFFECT*†

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Abstract

In this paper, we present a stability analysis of a Lotka-Volterra commensal symbiosis model subject to Allee effect on the unaffected population which occurs at low population density. By analyzing the Jacobian matrix about the positive equilibrium, we show that the positive equilibrium is locally asymptotically stable. By applying the differential inequality theory, we show that the system is permanent, consequently, the boundary equilibria of the system is unstable. Finally, by using the Dulac criterion, we show that the positive equilibrium is globally stable. Although Allee effect has no influence on the final densities of the predator and prey species, numeric simulations show that the system subject to an Allee effect takes much longer time to reach its stable steady-state solution, in this sense that Allee effect has unstable effect on the system, however, such an effect is controllable. Such an finding is greatly different to that of the predator-prey model.

Keywords Lotka-Volterra commensal symbiosis model; Allee effect; global stability

2000 Mathematics Subject Classification 34C25; 92D25; 34D20; 34D40

1 Introduction

Commensalism is one of the basic interrelationships between the species, where a species inflicts advantage to the other species without any harm or benefits received by the other. Epiphyte and plants with epiphyte, squirrel and the oak, etc, are the common typical commensal relationship, for more background about commensalism, one could see [1-5] and the references cited therein.

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Recently, Sun and Wei [6] proposed the following two species commensalism model:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= r_1 x\Big(\frac{k_1 - x + ay}{k_1}\Big), \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= r_2 y\Big(\frac{k_2 - y}{k_2}\Big), \end{aligned} \tag{1.1}$$

where r_1, r_2, k_1, k_2 and a are all positive constants. The system admits four equilibria:

$$A_1(0,0), \quad B_1(k_1,0), \quad C_1(0,k_2), \quad D_1(k_1+ak_2,k_2).$$

By linearizing the system at equilibrium, the authors investigated the stability of the above equilibrium points, and obtained the following results.

Theorem A

(1) $A_1(0,0))$ *is a unstable node;*

(2) $B_1(k_1,0)$ *is a saddle point;*

(3) $C_1(0,k_2)$ *is a saddle point;*

(4) $D_1(k_1+ak_2,k_2)$ *is a stable node.*

But we notice that the author did not give any information about the global stability property of the equilibrium. Based on (1.1), the scholars in [1-5,7-13] proposed varies commensalism models, and many interesting results were obtained, which have greatly enriched and improved the commensalism models during the last decade. For example: The dynamic behaviors of an impulsive periodic Lotka-Volterra commensal symbiosis model was studied in [1]; a two species commensal symbiosis model with Holling type functional response was investigated in [2]; a nonautonomous discrete two-species Lotka-Volterra commensalism system with delays was considered in [4]; sufficient conditions were obtained for the existence of positive periodic solution of a discrete commensal symbiosis model with Holling II functional response in [10]; a two-species commensal symbiosis model with ratio-dependent functional response was proposed in [11], etc. However, to this day, to the best of the authors knowledge, still no scholars consider the influence of Allee effect on the commensalism model.

Defined as a reduce per-capita population growth rate at low densities, Allee effect can be caused by difficulties in finding a mate or predator avoid danger or defense [14, 15]. It is well known that Allee effect has important consequences for population dynamics. In recent years, there are many papers concerned with Allee effect of the predator-prey system and competition system, etc, see [16-22] and the references cited therein. Their studies demonstrated that Allee effect can have important consequences on local stability property of the equilibrium, the equilibrium solution may be changed from stable to unstable or vice versa. However, the papers