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# PARALLEL COMPUTING METHOD OF PURE ALTERNATIVE SEGMENT EXPLICIT-IMPLICIT DIFFERENCE SCHEME FOR NONLINEAR LELAND EQUATION*†

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**Abstract**

The research on the numerical solution of the nonlinear Leland equation has important theoretical significance and practical value. To solve nonlinear Leland equation, this paper offers a class of difference schemes with parallel nature which are pure alternative segment explicit-implicit (PASE-I) and implicit-explicit (PASI-E) schemes. It also gives the existence and uniqueness, the stability and the error estimate of numerical solutions for the parallel difference schemes. Theoretical analysis demonstrates that PASE-I and PASI-E schemes have obvious parallelism, unconditionally stability and second-order convergence in both space and time. The numerical experiments verify that the calculation accuracy of PASE-I and PASI-E schemes are better than that of the existing alternating segment Crank-Nicolson scheme, alternating segment explicit-implicit and implicit-explicit schemes. The speedup of PASE-I scheme is 9.89, compared to classical Crank-Nicolson scheme. Thus the schemes given by this paper are high efficient and practical for solving the nonlinear Leland equation.

**Keywords** nonlinear Leland equation; pure alternative segment explicit-implicit scheme (PASE-I); stability; truncation error analysis; parallel computing; numerical experiments

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## 1 Introduction

In financial engineering and the modern finance, the most creative work is undoubtedly partial differential equations of option price which was derived by Black-

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Scholes and Merton in 1973. The result is a milestone in the history of financial derivative securities and sets a foundation for the reasonable pricing of various derivatives of the emerging derivative markets. The innovation of the model is that the option price does not depend on the personal preference of investors, but on the Black-Scholes model under too many assumptions which are inconsistent with the actual situation, such as no taxes and transaction fee, no arbitrage opportunities, et al. [1-4]. Therefore, the nonlinear Black-Scholes model has been the focus of academic research in the last 20 years.

Taking into account of the effect of payment transaction costs, Leland [5] improved the Black-Scholes model. Then Hoggard, Whalley and Wilmott [6] obtained option pricing formula under the transaction costs. Since it is almost impossible to find the exact analytic solution of the nonlinear Black-Scholes model [7], the numerical solution of the nonlinear Black-Scholes equation is of practical financial significance. Company et al. [8,9] gave a semi-discrete solution of the nonlinear Black-Scholes equation, and proved the consistency and stability of the numerical scheme. Pascal [10] gave an implicit numerical scheme for solving nonlinear option pricing models. When numerically solving multidimensional Black-Scholes equation or calculating the number of grid points, the required computation time will be exponentially increased and the computational efficiency will be declined. Thus the main problem here is that it is difficult to meet the requirements of the timeliness in the option pricing.

Along with the rapid development of high performance computer as well as the application of multi-core and cluster technology, parallel numerical method has turned into a new branch of science. It enriches the content of the traditional calculation methods and becomes an important research direction of computational mathematics. For parallel numerical methods, Evans and Abdullan [11] proposed grouping explicit ideas. For the implicit scheme, it has good stability but is not suitable to be solved in parallel. Inspired by grouping explicit methods, Zhang et al. [12,13] proposed the idea of constructing segment implicit using Saul'yev asymmetric scheme, and properly used the alternating technology to establish a variety of explicit-implicit and pure alternating implicit parallel method. The effect of stability and parallelism was obtained, but the accuracy was not very high due to the use of asymmetric scheme in the inner boundary points. Han et al. [14] constructed a class of pure alternating explicit implicit difference numerical methods, and proved the unconditional stability and high accuracy of the scheme for the diffusion equation. Zhang et al. [12,13] first named the most general explicit implicit hybrid scheme of the parabolic equation as difference scheme with parallel nature, then analyzed the theoretical issues such as the existence, uniqueness, convergence and stability