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THREE KIRCHHOFFIAN INDICES OF THE CACTUS GRAPHS*†

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Abstract

In this paper we give six explicit formulae to compute the Kirchhoff index, the multiplicative degree-Kirchhoff index and the additive degree-Kirchhoff index of the k-cactus chain and the cactus graph which can be obtained from a k-cactus chain by expanding each of the cut-vertices to a cut edge.

Keywords polyphenyl chain; cactus graph; Kirchhoff index; multiplicative degree-Kirchhoff index; additive degree-Kirchhoff index

2000 Mathematics Subject Classification 05C12

1 Introduction

The objects nowadays known as cactus appeared in the scientific literature more than half a century ago. Motivated by papers of Husimi [28] and Riddell [41], [44] dealt with cluster integrals in the theory of condensation in statistical mechanics. Besides statistical mechanics, where cacti and their generalizations serve as simplified models of real lattices [36, 42], the concept has also found applications in the theory of electrical and communication networks [56] and in chemistry [25,55]. Many topological indices have been studied for these structures, including the matching and independence polynomials [4, 16], the Hosoya indices [1],π-electron energy [52], the Hosoya polynomials [32], and the subtree numbers [50].

A *cactus graph* G is a connected graph in which each edge lies on at most one cycle. Therefore, each block in G is either an edge or a cycle. A *k-cactus* is a cactus in which each block is a k-cycle. A *k-cactus chain* is a k-cactus in which each block contains at most two cut-vertices and each cut-vertex lies in exactly two blocks. The number of blocks in a k-cactus chain is the length of the chain. A 6-cactus chain is also called spiro hexagonal chain, and a polyphenyl chain is a cactus graph which

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can be obtained from a 6-cactus chain by expanding each of the cut-vertices to an cut edge. For example, see the first graph in Figure 1.

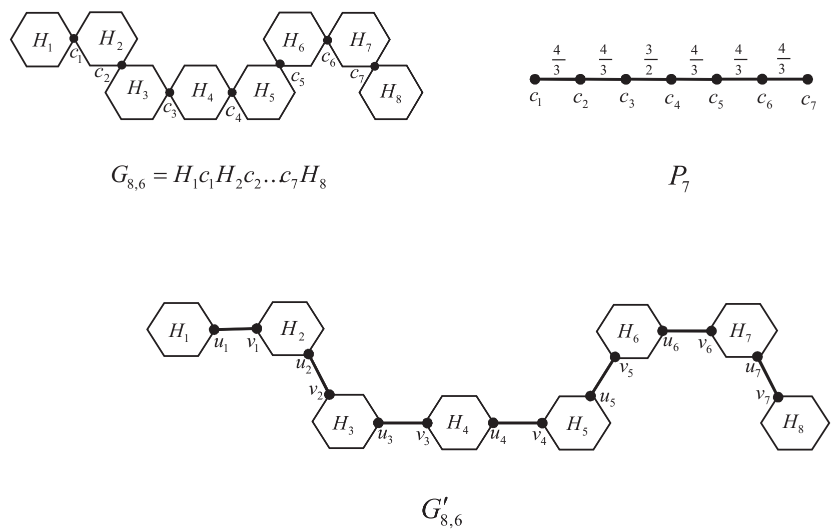

Figure 1: A spiro hexagonal chain, its corresponding weighted path and polyphenyl chain

Let G be a connected graph. The vertex set and edge set of G are denoted by $V(G)$ and $E(G)$, respectively. Based on the theory of electrical networks, Klein and Randić [30] introduced a new distance function named resistance distance. The resistance distance between a pair of vertices u and v in G, denoted by $r_G(u,v)$ or $r(u,v)$, is the effective resistance between them in the electrical network N constructed from G by replacing each edge with a unit resistor. This new intrinsic graph metric has being recognized as having more nice purely mathematical, chemical and physical interpretations [7, 12, 29–31].

Analogous to distance-based graph invariants, various graph invariants based on resistance distance have been defined and studied. Among these invariants, the most famous one is the Kirchhoff index [30], also known as the total effective resistance [21] or the effective graph resistance [18]. Like many topological indices, the Kirchhoff index is a structure descriptor and has been found very useful in purely mathematical, physical and chemical interpretations [30, 31, 54]. If the ordinary distance is replaced by the resistance distance in the expression for the Wiener index [47], one arrives at the Kirchhoff index [30].

Definition 1.1 The *Kirchhoff index* of a graph G is denoted by $Kf(G)$ and defined as follows:

$$Kf(G) = \sum_{\{u,v\}\subset V(G)} r_G(u,v).$$