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A COMMENSAL SYMBIOSIS MODEL WITH HOLLING TYPE FUNCTIONAL RESPONSE AND NON-SELECTIVE HARVESTING IN A PARTIAL CLOSURE*†

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Abstract

A two species commensal symbiosis model with Holling type functional response and non-selective harvesting in a partial closure is considered. Local and global stability property of the equilibria are investigated. Depending on the the area available for capture, we show that the system maybe extinct or one of the species will be driven to extinction, while the rest one is permanent, or both of the species coexist in a stable state. The dynamic behaviors of the system is complicated and sensitive to the fraction of the harvesting area.

Keywords commensal symbiosis model; stability; non-selective harvesting

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1 Introduction

During the last decade, many scholars have focused on the study of the commensal symbiosis model [1-12], which describes a relationship only favorable to the one side and has no influence to the other side.

Sun and Wei [4] first proposed a intraspecific commensal model:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= r_1 x\Big(\frac{k_1 - x + ay}{k_1}\Big), \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= r_2 y\Big(\frac{k_2 - y}{k_2}\Big). \end{aligned} \tag{1.1}$$

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They investigated the local stability of all equilibrium points. However, they did not give any information about the global dynamic behaviors of the system. Zhu, Xu and Li [11] proposed a commensalism model, in which one of the species could not be survived independently. By giving the phase trajectories analysis of the system, they are able to analyse the stability property of all the equilibria of the system. Han and Chen [5] incorporated the feedback control variables into the commensal symbiosis model system (1.1). By constructing some suitable Lyapunov function, they showed that the system admits a unique globally stable positive equilibrium; Xie et al. [6], Xue et al. [13,15] studied the dynamic behaviors of the discrete commensal symbiosis model; Miao et al. [16] further studied the dynamic behaviors of the periodic Lotka-Volterra commensal symbiosis model with impulse.

One could see that all of the above systems are based on the traditional Lotka-Volterra model, which made the assumption that the influence of the second species to the first one is linearize. This may not be suitable. Recently, Wu, Li and Zhou [1] further assumed that the relationship between two species is of Holling type functional response, and they established the following two species commensal symbiosis model

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= x\Big(a_1 - b_1 x + \frac{c_1 y^p}{1+y^p}\Big), \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= y(a_2 - b_2 y), \end{aligned} \tag{1.2}$$

where a_i, b_i, $i = 1, 2$, p and c_1 are all positive constants with $p \geq 1$. By applying the Dulac criterion, they showed that the unique positive equilibrium of the system is globally asymptotically stable.

It brings to our attention that all of the above study did not consider the influence of harvesting. Indeed, to obtain the resource for the development of the human being, harvesting of the species is necessary. Stimulated by the works of Chakraborty, Das and Kar [13], we further incorporated the non-selective harvesting term into system (1.2), and this leads to the following system

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= x\Big(a_1 - b_1 x + \frac{c_1 y^p}{1+y^p}\Big) - q_1 Emx, \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= y(a_2 - b_2 y) - q_2 Emy, \end{aligned} \tag{1.3}$$

where a_i, b_i, q_i, $i = 1, 2$, p, E and c_1 are all positive constants with $p \geq 1$ and E being the combined fishing effort used to harvest, and m $(0 < m < 1)$ is the fraction of the area which is available for harvesting.

The aim of this paper is to investigate the local and global stability property of the possible equilibria of system (1.3). We arrange the paper as follows: In the next