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Spatiotemporal Dynamics in a Generalized Diffusive Population System of Natural Pinus Koraiensis with Time Delay

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Abstract. In this paper, we establish a generalized population system of natural pinus koraiensis with lactation delay and diffusion term. Firstly, through the eigenvalue analysis, the conditions for local asymptotic stability of the positive equilibrium are derived, and the time delay is taken as bifurcation parameter, the existence conditions of Hopf bifurcation are discussed. Secondly, the model is analyzed qualitatively from the bifurcation point of view. The existence conditions of Turing bifurcation are given. By utilizing the normal form and center manifold theories of partial functional differential equations, the direction of Hopf bifurcation and the stability of bifurcating periodic solutions are studied, and the related formulae are determined. Finally, a nonlinear population model of pinus koraiensis with time delay and diffusion term is established. The corresponding numerical simulations are performed to verify the effects of time delay and diffusion on the stability of system, and the biological explanation is given.

AMS subject classifications: 34K18, 35B32

Key words: Population of pinus koraiensis, lactation delay, diffusion term, bifurcation, stability.

1 Introduction

1.1 Development of the population model of pinus koraiensis

The species value of pinus koraiensis is not only reflected in the protection of ecological environment, but also in the economy [1, 2]. Therefore, it is of great significance to study the ecosystem of pinus koraiensis. Due to the excessive cutting of pinus koraiensis, the number of resources decreased sharply, threatening the survival of pinus koraiensis.

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Therefore, for protecting biodiversity and maintaining natural ecological environment, it is necessary to establish a dynamic pinus koraiensis population model to simulate the dynamic process of population replacement in natural pinus koraiensis forest, analyze the population regeneration characteristics of pinus koraiensis and natural ecological process.

Since the mid-twentieth century, mathematical models were widely used to describe the state and dynamic characteristics of forests [3–6]. With the deterioration of ecological environment, the models that have been studied to explain the ecological environment were widely applied [7,8]. In 1988, Li Junqing et al. [9] studied a dynamic mathematical model of pinus koraiensis population simulating pine seeds, squirrels and seedlings. In 1994, Song Guohua studied a mathematical model of population agereplace of pines koraiensis in natural forest [10–13], which made the study on the population model of pinus koraiensis more practical. In 1996, in order to study the solution of model, Li Xiuqin [14] established a three-dimensional ecological mathematical model. Since then, a series of population models of pinus koraiensis that simulate seeds, squirrels and seedlings appeared, including linear, nonlinear, abundant year, deficient year and artificial intervention population models of pinus koraiensis [15–21].

1.2 Establishment of the model

In natural environment, due to the limited resources, the distribution of population is uneven in space, the organisms search for food to survive, then migration, diffusion will occur. Therefore, considering the inhomogeneity of the population distribution, the corresponding reaction-diffusion system is obtained. In addition, the change of population quantity is not only related to the current state, but also depends on the previous state, so the reaction-diffusion system with time delay has attracted many scholars [22–24].

We generalize the dynamic model of seeds, squirrels and pinus koraiensis seedlings, and introduce diffusion term and lactation delay of squirrels, the model satisfies homogeneous Neumann boundary conditions. Based on the following assumptions:

1. Ignoring other interfering factors, the pine seeds grew exponentially according to the ideal autotrophic development process. The main factor affecting the change of seed quantity is the size of squirrel population, which is decreasing under the influence of squirrels;

2. The quantity change of squirrel population is restricted by the seed yield and population density. The number of squirrels increases with the increase of pine seed quantity, and decreases with their own decline;

3. The population change of pinus koraiensis seedlings is affected by the caching behavior of squirrels. The predation and burial abilities of squirrels are the same. The seedlings increase with the effect of squirrels burying pine seeds, but decrease with their own thinning process.