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New Class of Kirchhoff Type Equations with Kelvin-Voigt Damping and General Nonlinearity: Local Existence and Blow-up in Solutions

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Abstract. In this paper, we consider a class of Kirchhoff equation, in the presence of a Kelvin-Voigt type damping and a source term of general nonlinearity forms. Where the studied equation is given as follows

$$u_{tt}-\mathcal{K}(\mathcal{N}u(t))\left[\Delta_{p(x)}u+\Delta_{r(x)}u_t\right]=\mathcal{F}(x,t,u).$$

Here, $\mathcal{K}(\mathcal{N}u(t))$ is a Kirchhoff function, $\Delta_{r(x)}u_t$ represent a Kelvin-Voigt strong damping term, and $\mathcal{F}(x,t,u)$ is a source term. According to an appropriate assumption, we obtain the local existence of the weak solutions by applying the Galerkin's approximation method. Furthermore, we prove a non-global existence result for certain solutions with negative/positive initial energy. More precisely, our aim is to find a sufficient conditions for $p(x),q(x),r(x),\mathcal{F}(x,t,u)$ and the initial data for which the blow-up occurs.

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1 Introduction

1.1 Statement of the problem

The problem with variable exponents occurs in many mathematical models of applied science, for example, viscoelastic fluids, electro rheological fluids, processes of filtration through a porous media, fluids with temperature-dependent viscosity etc. From the mathematical point of view, the question of existence, uniqueness and behavior of solutions remain essential results to describe various phenomena. The blow-up is one of the most important behaviors that have been dealt with in evolution problems. In this article, we consider the following initial boundary value problem

$$\begin{cases} u_{tt}-\mathcal{K}(\mathcal{N}u(t))\left[\Delta_{p(x)}u+\Delta_{r(x)}u_t\right]=\mathcal{F}(x,t,u), & (x,t)\in Q_T=\Omega\times(0,T), \\ u(x,t)=0, & x\in\partial\Omega,\, t\in(0,T), \\ u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), & x\in\Omega, \end{cases} \tag{1.1}$$

where Ω is an open bounded Lipschitz domain in $\mathbb{R}^N\,(N\geq 1)$ with smooth boundary $\partial\Omega$, $T\in(0,+\infty]$ is the maximal existence time of the solutions $u(x,t)$. The initial conditions fulfill the following

$$u_0\in W_0^{1,p(\cdot)}(\Omega) \quad \text{and} \quad u_1\in L^2(\Omega). \tag{1.2}$$

The Kirchhoff function $\mathcal{K}\in C(\mathbb{R}^+,\mathbb{R}^+)$ takes the form

$$\mathcal{K}(\tau)=a+b\gamma\tau^{\gamma-1}, \quad a,b\geq 0,\ \gamma\geq 1,\ a+b>0,\ \gamma>1\ \text{ if }\ b>0. \tag{1.3}$$

The elliptic nonhomogeneous $s(x)-$Laplacian operator is defined by

$$\Delta_{s(x)}u=\nabla\cdot(|\nabla u|^{s(x)-2}\nabla u),$$

where $\nabla\cdot$ is the vectorial divergence and ∇ the gradient. This operator can be extended to a monotone operator between the spaces $W_0^{1,s(\cdot)}(\Omega)$ and its dual as follows

$$\begin{cases} -\Delta_{s(\cdot)}:W_0^{1,s(\cdot)}(\Omega)\to W^{-1,s'(\cdot)}(\Omega), \\ \left\langle -\Delta_{s(\cdot)}u,\phi\right\rangle_{s(\cdot)}=\int_\Omega |\nabla u|^{s(x)-2}\nabla u\cdot\nabla\phi \mathrm{d}x, \quad 2\leq s(x)<\infty, \end{cases}$$

where $\langle\cdot,\cdot\rangle_{s(\cdot)}$ denotes the duality pairing between $W^{-1,s'(\cdot)}(\Omega)$ and $W_0^{1,s(\cdot)}(\Omega)$ with $\frac{1}{s(\cdot)}+\frac{1}{s'(\cdot)}=1$.

We consider $\mathcal{N}$ as the $p(x)$-Dirichlet integral defined by

$$u\longmapsto\int_\Omega \frac{|\nabla u|^{p(x)}}{p(x)}\mathrm{d}x.$$

We assume that the nonlinearity $\mathcal{F}(x,t,u)$ in (1.1) satisfy the following two assumptions: