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An Edge-Based Smoothed Finite Element Method with TBC for the Elastic Wave Scattering by an Obstacle

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Abstract. Elastic wave scattering has received ever-increasing attention in military and medical fields due to its high-precision solution. In this paper, an edge-based smoothed finite element method (ES-FEM) combined with the transparent boundary condition (TBC) is proposed to solve the elastic wave scattering problem by a rigid obstacle with smooth surface, which is embedded in an isotropic and homogeneous elastic medium in two dimensions. The elastic wave scattering problem satisfies Helmholtz equations with coupled boundary conditions obtained by Helmholtz decomposition. Firstly, the TBC of the elastic wave scattering is constructed by using the analytical solution to Helmholtz equations, which can truncate the boundary value problem (BVP) in an unbounded domain into the BVP in a bounded domain. Then the formulations of ES-FEM with the TBC are derived for Helmholtz equations with coupled boundary conditions. Finally, several numerical examples illustrate that the proposed ES-FEM with the TBC (ES-FEM-TBC) can work effectively and obtain more stable and accurate solution than the standard FEM with the TBC (FEM-TBC) for the elastic wave scattering problem.

AMS subject classifications: 35Q99, 65N99

Key words: Elastic wave scattering problem, edge-based smoothed finite element method, Helmholtz equations, transparent boundary condition.

1 Introduction

The obstacle scattering is a basis issue of scattering theory, which considers the incident wave scattering by an impenetrable and bounded media. It plays an increasingly important role in many scientific and engineering fields, such as target detection and positioning in radar and sonar; imaging of single proteins in medical imaging and oil and gas

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exploration in geophysical exploration [1]. Common scattering problem mainly include: acoustic scattering, elastic wave scattering and electromagnetic wave scattering. Compare to acoustic and electromagnetic wave scattering [2–4], the elastic wave scattering is less studied because of the existence of compressional and shear waves, which will increase computational difficulty. However, because the elastic wave scattering can obtain more accurate solution, it has recently attracted more and more attention in geophysics and seismology [5–10]. Meanwhile, the existence and uniqueness for solution to the elastic scattering problem have been demonstrated in Refs. [11, 12]. The analytical solution has been usually obtained for the wave scattering with simple geometrical obstacle [13], while it is difficult to obtain the analytical solution for obstacles with relatively complicated shapes.

Therefore, to tackle the wave scattering problem by more complicate obstacle, some numerical methods have been proposed for solving direct and inverse obstacle scattering [2,14], such as finite element method (FEM) [3,15–17], boundary integral method [18] and dynamical functional particle method (DFPM) [19]. Among these numerical methods, the FEM is a widely used and powerful numerical method. Considering that the elastic wave scattering is described as an exterior boundary value problem imposed in an unbounded domain, people usually truncate the unbounded domain into a bounded domain before using the FEM solves the problem. There are two common truncation techniques: one is the transparent boundary condition (TBC), the other is a technique called the perfectly matched layer (PML). The TBC was firstly proposed based on a non-local Dirichlet-to-Neumann (DtN) operator, which is defined by an infinite Fourier series. By imposing the TBC on the boundary of the truncated domain, the artificial wave reflection can not occur. The TBC has been applied to work out some wave scattering problems [2, 14, 20–24]. The PML technique was proposed by Berenger [25], which uses an artificial layer to truncate the computational region. The layer can guarantee that the scattered wave is not reflected from the PML medium. The application of the PML can be founded in researches [3, 26–28].

Though the FEM combined with the TBC or PML can handle the elastic wave scattering problem, the accuracy of the solution is not particularly high due to over-stiff characteristic of the FEM. To avoid over-stiff drawback and improve the accuracy of the solution, Liu et al. established the G space theory and proposed the weakened weak (W2) formulations [29–31] for some numerical methods, e.g. smoothed finite element method (S-FEM) [32–36] and smoothed point interpolation method (S-PIM) [37–39]. The S-FEM can soften over-stiff behavior of the FEM, hence the method has been extensively employed in various problem, such as wave problem [40–46], contact problem [47, 48], solid mechanics problem [12, 49–51] and fracture problem [53, 54]. In general, the S-FEM model can be classified into different smoothed finite element models based on different types of the smoothing domain. There are four common models, which are a cell-based smoothed finite element method (CS-FEM), a node-based smoothed finite element method (NS-FEM), an edge-based smoothed finite element method (ES-FEM) and a face-based smoothed finite element method (FS-FEM). In these common models, the ES-FEM