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Uniform Convergence of Spectral Expansions in the Terms of Root Functions of a Spectral Problem for the Equation of a Vibrating Beam

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Abstract. In this paper we consider a spectral problem which describes bending vibrations of a homogeneous rod, in cross-sections of which the longitudinal force acts, the left end of which is fixed rigidly and on the right end is concentrated an elastically fixed load. We study the uniform convergence of spectral expansions in terms of root functions of this problem.

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1 Introduction

The small bending vibrations of a homogeneous rod (Euler-Bernoulli beam) of length L and of constant rigidity, in cross sections of which the longitudinal force acts, is described by the equation [7, Ch. 8, Section 5, formula (84)]

$$EJ\frac{\partial^4 U(X,t)}{\partial X^4}-\frac{\partial}{\partial X}\left(\tilde{Q}(X)\frac{\partial U(X,t)}{\partial X}\right)+\rho F\frac{\partial^2 U(X,t)}{\partial t^2}=0,$$

where $U(X,t)$ is a flexure of the current point of axis of the rod, EJ is the flexural rigidity of the rod, $\tilde{Q}(X)$ is longitudinal force.

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If the left end is fixed rigidly and at the right end is concentrated an elastically fixed load with mass of m, then the boundary conditions can be written in the following form [7, Ch. 8, Section 5, p. 154]

$$U(0,t)=0, \qquad \frac{\partial U(0,t)}{\partial X}=0, \qquad EJ\frac{\partial^2 U(L,t)}{\partial X^2}=0,$$
$$EJ\frac{\partial^3 U(L,t)}{\partial X^3}-\tilde{Q}(L)\frac{\partial U(L,t)}{\partial X}+c_1 U(L,t)=-m\frac{\partial^2 U(L,t)}{\partial t^2}.$$

By use of the notation $x=\frac{X}{L}$, $u=\frac{U}{L}$ we can write the equation of small bending vibrations of a homogeneous rod and the boundary conditions in the following form:

$$\frac{\partial^4 u(x,t)}{\partial x^4}-\frac{\partial}{\partial x}\left(Q(x)\frac{\partial u(x,t)}{\partial x}\right)+\frac{\rho F L^4}{EJ}\frac{\partial^2 u(x,t)}{\partial t^2}=0,$$
$$u(0,t)=0, \qquad \frac{\partial u(0,t)}{\partial x}=0, \qquad \frac{\partial^2 u(1,t)}{\partial x^2}=0,$$
$$\frac{\partial^3 u(1,t)}{\partial x^3}-Q(1)\frac{\partial u(1,t)}{\partial x}+\frac{c_1 L^3}{EJ}u(1,t)=-\frac{mL^3}{EJ}\frac{\partial^2 u(1,t)}{\partial t^2},$$

where $Q(x)=\frac{L^2}{EJ}\tilde{Q}(Lx)$.

Let $\lambda=\rho F L^4\omega^2/EJ$. Then considered problem with substitution $u(x,t)=y(x)\cos\omega t$ is reduced (see, e.g., [7, Ch. 11, Section 2, formula (12)]) to the following eigenvalue problem

$$y^{(4)}(x)-(q(x)y'(x))'=\lambda y(x), \quad 0<x<1, \tag{1.1a}$$
$$U_1(\lambda,y)\equiv y(0)=0, \quad U_2(\lambda,y)\equiv y'(0)=0, \quad U_3(\lambda,y)\equiv y''(1)=0, \tag{1.1b}$$
$$U_4(\lambda,y)\equiv Ty(1)-(a\lambda+b)y(1)=0, \tag{1.1c}$$

where

$$q(x)\equiv Q(x)>0, \quad x\in[0,1], \quad Ty\equiv y'''-qy', \quad a=\frac{m}{\rho FL}>0, \quad b=-\frac{c_1L^3}{EJ}<0.$$

Moreover, we assume that $q(x)$ is an absolutely continuous function on $[0,1]$.

One of the most common methods for solving partial differential equations is method of separation of variables. The justification of this method is based on the convergence of spectral expansions in the systems of root functions of the corresponding eigenvalue problems in various functional spaces.

The spectral properties of problem (1.1a)-(1.1c) in more general form were investigated in the paper [2]. The subject of the present paper is the study of uniform convergence of Fourier series expansions for continuous functions in the subsystems of root functions of problem (1.1a)-(1.1c).