JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **34**, No. 2, pp. 129-143

doi: 10.4208/jpde.v34.n2.3
June 2021

Asymptotic Behavior in a Quasilinear Fully Parabolic Chemotaxis System with Indirect Signal Production and Logistic Source

LI Dan and LI Zhongping*

College of Mathematic and Information, China West Normal University, Nanchong 637009, China.

Received 4 December 2020; Accepted 5 January 2021

Abstract. In this paper, we study the asymptotic behavior of solutions to a quasilinear fully parabolic chemotaxis system with indirect signal production and logistic source

$$\begin{cases} u_t=\nabla\cdot(D(u)\nabla u)-\nabla\cdot(S(u)\nabla v)+b-\mu u^{\gamma}, & x\in\Omega,\ t>0,\\ v_t=\Delta v-a_1v+b_1w, & x\in\Omega,\ t>0,\\ w_t=\Delta w-a_2w+b_2u, & x\in\Omega,\ t>0 \end{cases}$$

under homogeneous Neumann boundary conditions in a smooth bounded domain $\Omega\subset\mathbb{R}^n$ $(n\geq 1)$, where $b\geq 0$, $\gamma\geq 1$, $a_i\geq 1$, $\mu,b_i>0$ $(i=1,2)$, $D,S\in C^2([0,\infty))$ fulfilling $D(s)\geq a_0(s+1)^{-\alpha}$, $0\leq S(s)\leq b_0(s+1)^{\beta}$ for all $s\geq 0$, where $a_0,b_0>0$ and $\alpha,\beta\in\mathbb{R}$ are constants. The purpose of this paper is to prove that if $b\geq 0$ and $\mu>0$ sufficiently large, the globally bounded solution (u,v,w) with nonnegative initial data (u_0,v_0,w_0) satisfies

$$\left\|u(\cdot,t)-\left(\frac{b}{\mu}\right)^{\frac{1}{\gamma}}\right\|_{L^\infty(\Omega)}+\left\|v(\cdot,t)-\frac{b_1b_2}{a_1a_2}\left(\frac{b}{\mu}\right)^{\frac{1}{\gamma}}\right\|_{L^\infty(\Omega)}+\left\|w(\cdot,t)-\frac{b_2}{a_2}\left(\frac{b}{\mu}\right)^{\frac{1}{\gamma}}\right\|_{L^\infty(\Omega)}\to 0$$

as $t\to\infty$.

AMS Subject Classifications: 35K55, 35Q92, 35Q35, 92C17

Chinese Library Classifications: O175.29

Key Words: Chemotaxis system; indirect signal; logistic source; asymptotic behavior.

---

*Corresponding author. *Email addresses:* 1923817761@qq.com (D. Li), zhongpingli80@126.com (Z. P. Li)

1 Introduction

In 1970, Keller and Segel proposed a classical biological chemotaxis model [1]

$$\begin{cases} u_t=\Delta u-\nabla\cdot(u\nabla v), & x\in\Omega,\ t>0,\\ v_t=\Delta v-v+u, & x\in\Omega,\ t>0,\\ \dfrac{\partial u}{\partial \nu}=\dfrac{\partial v}{\partial \nu}=0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x),\quad v(x,0)=v_0(x), & x\in\Omega \end{cases} \tag{1.1}$$

with u represents the density of cells, v is the density of a chemical signal. In the first equation of (1.1), Δu denotes self diffusion and the cross-diffusion term $-\nabla\cdot(u\nabla v)$ means that the cell is moving towards a high chemical concentration. In the second equation of (1.1), Δv is the self-diffusion of the chemical signal, $-v+u$ denotes the consumption of v and direct production by the cell u. The system (1.1) describes the chemotactic behavior of cells in numerous biological processes [2,3], and the biological model (1.1) plays a key role. When $n=1$, the system (1.1) has a unique global solution [4]. When $n=2$, there is a critical mass phenomenon [5], if $\int_\Omega u_0<4\pi$, the system (1.1) processes a globally bounded classical solution; if $\int_\Omega u_0>4\pi$, the solution of the system (1.1) will blow up [6]. When $n\geq 3$, if Ω is a ball, then for arbitrarily small mass $m:=\int_\Omega u_0>0$, there exists (u_0,v_0) such that (u,v) blowing up [7].

Ever since 1970, mathematicians have intensively investigated different types of chemotaxis models for a variety of chemotaxis processes [2]. When considering the logistic source, some researchers studied the corresponding quasilinear chemotaxis system of (1.1)

$$\begin{cases} u_t=\nabla\cdot(D(u)\nabla u)-\nabla\cdot(S(u)\nabla v)+f(u), & x\in\Omega,\ t>0,\\ v_t=\Delta v-v+u, & x\in\Omega,\ t>0,\\ \dfrac{\partial u}{\partial \nu}=\dfrac{\partial v}{\partial \nu}=0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x),\quad v(x,0)=v_0(x), & x\in\Omega \end{cases} \tag{1.2}$$

with $D,S\in C^2([0,\infty))$ satisfying $D(s)\geq c_0 s^p$, $c_1 s^q\leq S(s)\leq c_2 s^q$. When $f\equiv 0$, previous works have investigated whether the solutions are globally bounded or blow up [8–10]. It is well-known that logistic sources favor the existence of the globally bounded solution. Indeed, if $f(s)$ is a smooth function fulfilling $f(0)\geq 0$ and $f(s)\leq as-\mu s^2$ for all $s>0$, it is shown that whenever $q<1$, there exists a unique globally bounded and classical solution [11, 12]. When $f(s)\leq a-\mu s^2$ for all $s\geq 0$, with $a\geq 0$ and $\mu>0$ properly large, if $n\geq 3$ and Ω is convex, Winkler [13] showed the global boundedness of solutions. The chemotaxis signal of (1.2) is produced directly by cells, yet the signal generation undergoes intermediate stages in some realistic biological processes [14–16], the indirect signal production mechanism can cause different interaction of cross-diffusion and the logistic