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Risk Measure and Premium Distribution on Catastrophe Reinsurance*

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Abstract: In this paper, we propose a new risk measure which is based on the Orlicz premium principle to characterize catastrophe risk premium. The intention is to develop a formulation strategy for Catastrophe Fund. The logarithm equivalent form of reinsurance premium is regarded as the retention of reinsurer, and the differential earnings between the reinsurance premium and the reinsurer's retention is accumulated as a part of Catastrophe Fund. We demonstrate that the aforementioned risk measure has some good properties, which are further confirmed by numerical simulations in R environment.

Key words: catastrophe reinsurance, catastrophe fund, Orlicz premium principle, Haezendonck-Goovaerts risk measure, stochastic ordering

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1 Introduction

Insurance companies need reinsurance to limit their liabilities on catastrophe risk. Even if reinsurance prices are high relative to expected loss, risk-averse insurers are still willing to seek reinsurance protections against large events (see [1–2]). Actually, as Cummins *et al.* declared in [3], the enormous loss would stress the capacity of the insurance underwriting and threaten the credit risk of many reinsurance companies. Catastrophe Fund can improve the availability and affordability of the insurance industry by reimbursing the insured a portion of their catastrophic losses.

Most Catastrophe Fund comes from government program (see [4]). Following the assumption in [5–6], we suggest a Catastrophe fund that limits the taxpayer's exposure to the catastrophic disaster and reduces cross-subsidization of high-risk insurers by low-risk

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insurers in the reinsurance market.

Stop-loss reinsurance is the optimal contract in many cases (see, e.g., [7–8]). We use a new risk measure based on the Orlicz premium principle (see [9–10]) to determine the stop-loss reinsurance premium and then to propose a premium distribution scheme.

The new risk measure has some good properties such as monotone, translation invariance, convex-preserving, subadditivity and upper and lower boundedness, which are further confirmed by numerical simulations. Some concepts of stochastic order can be found in [11–14].

The rest of this paper is orgnalized as follows: in Section 2, we present the definition and some properties of the new risk measure. Section 3 compares the new risk measure with Haezendonck-Goovaerts risk measure and suggests the premium distribution scheme. Sections 4 describes the numerical simulation results. Section 5 gives the conclusions, and finally, Appendix contains some proofs.

2 A New Risk Measure Based on the Orlicz Premium Principle

For given risk variable $X \in L_{+}^{\infty}$ and $\alpha \in [0, 1)$, the Orlicz premium principle is given by the unique solution $H_\alpha(X)$ of the equation

$$E\Big[\varphi\Big(\frac{X}{H_\alpha(X)}\Big)\Big] = 1 - \alpha, \qquad X \neq 0,$$

where $\varphi(\,\cdot\,)$ is a non-negative, strictly increasing, and continuous function on $[0, +\infty)$ with $\varphi(0) = 0$, $\varphi(1) = 1$, $\varphi(+\infty) = +\infty$. In [15] the functional φ is described in detail, and in [16] we know that the φ is a convex function.

Denote by π an Orlicz risk measure on stop-loss reinsurance and x a retention. The equation of Orlicz premium is

$$E\Big[\varphi\Big(\frac{(X-x)_+}{\pi - x}\Big)\Big] = 1 - \alpha.$$

We modify the equation as follows:

$$E\Big[\varphi\Big(\frac{(X-x)_+}{\ln(1+\pi-x)}\Big)\Big] = 1 - \alpha. \tag{2.1}$$

Let X be a continuous type random variable with the distribution function $F(x)$, $-\infty \le \min\{X\} \le \max\{X\} \le +\infty$, $F_X^{-1}(\alpha) = \inf\{x : F(x) \ge \alpha\}$ be the α quantile of the risk X. $\pi_\alpha[X,\ x]$ represents the modified Orlicz risk measure of the risk X for the retention x at the level α. Then we have the following Theorem 2.1:

Theorem 2.1 *Let x be the retention, X, α and $\varphi(\,\cdot\,)$ are as the aforementioned. Then for any x, $-\infty < x < \max\{X\}$, and $\alpha \in (0, 1)$, the equation (2.1) has a unique solution $\pi_\alpha[X,\ x]$ satisfying the inequalities*

$$\pi_\alpha[X,\ x] \ge F_X^{-1}(\alpha), \qquad \pi_\alpha[X,\ x] > x.$$

Proof. Let

$$H(\pi) = E\Big[\varphi\Big(\frac{(X-x)_+}{\ln(1+\pi-x)}\Big)\Big].$$