Communications
in
Mathematical
Research
31(2)(2015), 141–148

Zeros Distribution of the Jones Polynomial for Pretzel Links

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Communicated by Lei Feng-chun

Abstract: In this paper, we deal with basic properties of some pretzel links and properties of the Jones polynomials of some pretzel links. By using these properties, the zero distribution of pretzel links is studied. We discuss the properties of the Jones polynomial of non-tame pretzel links and give that zeros of the Jones polynomial of these pretzel links are distributed on the planar curves.

Key words: Jones polynomial, zero, pretzel link
2010 MR subject classification: 57M15, 57M25
Document code: A
Article ID: 1674-5647(2015)02-0141-08
DOI: 10.13447/j.1674-5647.2015.02.05

1 Introduction

Invariants of knots play a very important role in classification of knots. The Alexander polynomial (see [1]) is a milestone in the knot theory, however it cannot distinguish a knot from its mirror. In 1984, Jones[2] found a new knot invariant (later, it is called Jones polynomial) and it is an ambient isotopy invariant, and the calculation is convenient. His findings made the knot theory known as one of the focus at the mathematical field in the world. Lin[3] discussed the properties of zeros of Jones polynomial. This is helpful to study the relations between the Laurent polynomials and Jones polynomial. In [4–5], the authors gave the Jones polynomial for the pretzel links $P(k,\ k,\ k)$, $P(\overbrace{3,\ 3,\ \cdots,\ 3}^{n})$, $P(k,\ \overbrace{1,\ \cdots,\ 1}^{n-1})$ and $P(k,\ \overbrace{2,\ \cdots,\ 2}^{n-1})$ $(k > 0)$ by the relations between Jones polynomial and some physical models. These are good to discuss zeros of Jones polynomial. Jin and Zhang[4] dealt with the zeros distribution of the pretzel links $P(k,\ k,\ k)$, $P(\overbrace{3,\ 3,\ \cdots,\ 3}^{n})$, $P(k,\ \overbrace{1,\ \cdots,\ 1}^{n-1})$ and

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Received date: Aug. 26, 2013.
Foundation item: The NSF (11071106) of China, and the Program (LR2011031) for Liaoning Excellent Talents in University.
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$P(k, \overbrace{2, \cdots, 2}^{n-1})$ $(k > 0)$. In [6–7], zeros of the Jones polynomial are discussed. In Section 2, we give some definitions needed in the paper and properties of the Jones polynomial of the pretzel links $P(\overbrace{k, k, \cdots, k}^{n})$ and $P(k, \overbrace{l, \cdots, l}^{n-1})$ $(k > 0,\ l > 0)$. Furthermore, we study zeros distribution of the Jones polynomials.

2 The Zeros of Jones Polynomial of the Pretzel Links

Definition 2.1 *A pretzel link* $P(c_1, c_2, \cdots, c_n)$ *is determined by an n-tuple* $(c_1, c_2, \cdots, c_n)$, *where* $c_i \neq 0$, $i = 1, 2, \cdots, n$, $n \geq 3$, *the absolute value of* c_i *gives the number of half twists, and the sign of* c_i *indicates either positive or negative half twists. The standard diagram is shown as Fig.* 2.1.

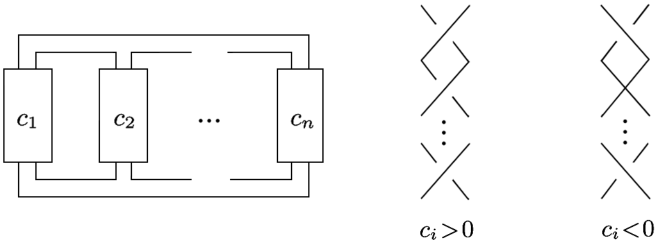

Fig 2.1

Lemma 2.1[4] (1) (i) *If all* c_i *are odd numbers, then* $P(c_1, c_2, \cdots, c_n)$ *is a knot when* n *is an odd number;* $P(c_1, c_2, \cdots, c_n)$ *is a link with two components when* n *is an even number;*

(ii) *If there exists some even* c_i, *then the number of even* c_i*'s equals to the number of components of* $P(c_1, c_2, \cdots, c_n)$*;*

(2) *If the signs of all the* c_i*'s are the same (i.e., they are either positive or negative), then* $P(c_1, c_2, \cdots, c_n)$ *is an alternating link;*

(i) *If* $c_i > 0$, *then the number of A-regions is*

$$\tilde{a} = 2 + \sum_{i=1}^{n}(c_i - 1),$$

and the number of B-regions is

$$\tilde{b} = n;$$

(ii) *If* $c_i < 0$, *then the number of A-regions is*

$$\tilde{a} = n,$$

and the number of B-regions is

$$\tilde{b} = 2 + \sum_{i=1}^{n}(c_i - 1).$$