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The Projected Newton Iteration Approach for Computing the Nonnegative Z-Eigenpairs of Nonnegative Tensors

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Abstract. In this paper, we propose a new projected Newton iteration for computing the nonnegative Z-eigenpairs of nonnegative tensors. We show that the required iteration has a local quadratic convergence. More specially, the formulation aims to solve the tensor equation arising from the multilinear PageRank problem. Numerical experiments are provided to illustrate the effectiveness and superiority of the proposed approach.

AMS subject classifications: 65F15, 65F50

Key words: Nonnegative tensor, nonnegative Z-eigenpair, local quadratic convergence, multilinear PageRank.

1 Introduction

The eigenvalue problems of tensors have received much attention in recent years because of their wide applications such as medical imaging [2,5], higher order Markov chains [14], blind source separation [9], etc. Unlike the matrix case, there are several definitions of tensor eigenvalues, e.g., H-eigenvalues [15,18], Z-eigenvalues [15,18], D-eigenvalues [19], based on the practical problems. In this paper, we focus on computing the Z-eigenvalues and the corresponding Z-eigenvectors by targeting on the applications to the PageRank problem.

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Currently, there are many proposed approaches in terms of iteration formulation in literature to compute one or more Z-eigenpairs of tensors with special structures, such as nonnegative tensors and symmetric tensors. The challenge is, even if a tensor is symmetric and/or nonnegative, that there may be more than one eigenpair. This leads to the impossibility for the global convergence in general. One of the notable methods for computing Z-eigenpairs of symmetric tensors is the so-called shifted power method (SS-HOPM) proposed by Kolda and Mayo [10]. The convergence rate of SS-HOPM depends on the choice of the shift. If the shift is not properly chosen, the method will converge slowly or even diverge in some occasions. To address this issue, Kolda and Mayo in [11] further added an adaptive procedure for choosing the shift. Most recently, Zhao et al. [20] proposed a modified normalized Newton method (MNNM) for computing Z-eigenpairs of symmetric tensors which can be convergent cubically. For the nonnegative tensors, Guo et al. [7] proposed a modified Newton iteration (MNI) to find some positive Z-eigenpairs and showed that their method has a local quadratic convergence under appropriate assumptions. In this paper, by reexamining the existing approaches, we develop a new approach via the projected Newton iteration for computing the nonnegative Z-eigenpairs of nonnegative tensors which improves the convergence rate of MNI.

The rest of the paper is organized as follows. In Section 2, we introduce some necessary notions as well as some preliminary results related to tensors, and provide a review for MNI. In Section 3, we propose a projected Newton iteration (PNI) for computing the nonnegative Z-eigenpairs of nonnegative tensors and show its local quadratic convergence. In Section 4, numerical experiments are provided to demonstrate the effectiveness and convergent behavior of the proposed PNI. In particular, we apply PNI to the multilinear PageRank. Finally, concluding remarks are given in Section 5.

2 Preliminaries

In this section, we recall some definitions and properties related to tensors, we also briefly review the MNI method for computing nonnegative Z-eigenpairs of nonnegative tensors in literature.

2.1 Notations and definitions

Let $\mathbb{R}$ be the real field, $\langle n\rangle=\{1,\cdots,n\}$ and $\mathcal{A}\in\mathbb{R}^{n\times\cdots\times n}$ be a tensor of order m and dimension n, with entries $\mathcal{A}_{i_1\cdots i_m}$, where $i_1,\cdots,i_m\in\langle n\rangle$. We say that $\mathcal{A}$ is nonnegative, if $\mathcal{A}_{i_1\cdots i_m}\geq 0$ for $i_1,\cdots,i_m\in\langle n\rangle$ element wise. Throughout this paper, we denote the set of all real nonnegative tensors of order m and dimension n by $\mathbb{R}_+^{[m,n]}$. $\mathcal{A}$ is called semisymmetric [17] if the value of its entries $\mathcal{A}_{i_1\cdots i_m}$ is invariant under any permutation of their indices $i_2,\cdots,i_m$.

For real vectors $\mathbf{v}=(v_1,\cdots,v_n)$ and $\mathbf{w}=(w_1,\cdots,w_n)$, we define the following element-