Journal of Nonlinear Modeling and Analysis
Volume 1, Number 4, December 2019, 443–459
http://jnma.ca; http://jnma.ijournal.cn
DOI:10.12150/jnma.2019.443

The Formats of Julia Sets for Complex Dynamic Systems

Changan Liu[1] and Shutang Liu[2,†]

Abstract In this paper, the formats of Julia sets for a class of nonlinear complex dynamic systems with variable coefficients were studied under certain conditions. For the complex dynamic systems in piecewise cases, we proposed some methods to control the forms of their Julia sets and stable domains analytically. What's more, we illustrated that our methods worked well by computational simulations. Our work provides a better understanding about how to control the Julia sets of certain complex dynamic systems.

Keywords Fractals, variable coefficients, complex dynamic system, stable domain, Julia sets.

MSC(2010) 28A80, 37F50.

1. Introduction

People began to learn the complex dynamic systems during World War I. Inspired by the method of Newton iterative operations and the ultimate sets of Mobius transform group, French mathematicians P. Fatou and G. Julia found some interesting results on Riemman sphere [8,12]. From 1918 to 1920, they applied the new theories of normal train (such as Montel theorem) on dynamic systems to prove a series of valuable results, which completing the fundamental work of complex dynamic systems, defined the famous fractal set-Julia set, and forming the classical Fatou-Julia theory. The Mandelbrot set [13] is highly related to the Julia set, which was defined in 1980 by Benoit B. Mandelbrot. It is the result of iterating the dynamic systems. Though the systems and the iterative operations are simple, the shapes and the fine structures of the results are shocking.

At present, the research of complex dynamic systems is still the focus, involving its qualitative theory [27] and the control of bounded domains for the fractal sets [26, 28]. In addition, it provides novel methods for studying all kinds of complex shapes and structures in the nature. So it is widely applied in astronomy [6, 24], geography [16, 18, 19, 23], physics [3, 10, 20, 22], chemistry [5, 11], biology [1, 9, 15, 17], materials [2, 14, 21, 25], sociology [4] and so on.

In particular, lots of problems involve the stability of systems in engineering and technology. The stability of systems relate to their stable domains, or the shapes and sizes of the stable areas. Julia sets and Mandelbrot sets can describe the the

[†]the corresponding author.
Email address: podlca@math.uh.edu(C. Liu), stliu@sdu.edu.cn(S. Liu)
[1]Department of Mathematics, University of Houston, 3551 Cullen Blvd., TX 77204-3008, Houston, United States
[2]College of Control Science and Engineering, Shandong University, Jinan, Shandong 250061, China

shapes of the stable fields for the systems. Here, we give the definition of the formats of complex dynamic systems and introduce Julia set as follows.

Definition 1.1. For the nonlinear time delay complex dynamic system:

$$z_{n+1} = f(a_n, c_n, z_n, z_{n-k}) \ , \tag{1.1}$$

where a_n and c_n are known sequences of complex numbers, k is a quantity of time delay and a nonnegative integer. The formats of complex dynamic systems are the process which makes the stable domain of system (1.1) to achieve some enacted aims by adjusting a_n, c_n and k.

Especially, when analyzing system (1.1) through the definition of Julia sets, the stable domains are the formats of Julia set of system (1.1).

For (1.1), let C is a set of all complex numbers, and we call $\partial(C)$ the boundary of C. The definition of Julia sets is as follows [7].

Definition 1.2. For complex polynomial f, let

$$\mathcal{K}(f) = \{ \ z \in C : f^k(z) \nrightarrow \infty, \ k \to \infty \ \} \ ,$$

$\mathcal{J}(f) \triangleq \partial(\mathcal{K}(f))$. We call $\mathcal{J}(f)$ the Julia set of system (1.1).

In this paper, firstly, we will study the formats of Julia sets for above system in general conditions. Then we analyze the formats of Julia sets of special conditions of the system above, which are generated through modifying the complex sequences a_n and c_n. At last, we study the formats of Julia sets for the system above in the piecewise cases. We will illustrate our results through simulations.

For (1.1), we consider that

$$f(\cdot) = a(z+b)^2 - c \ .$$

For the complex dynamic system:

$$z_{n+1} = a_n(z_n + c_n)^2 - c_{n+1} \ , \tag{1.2}$$

it becomes to the well-known classical system

$$z_{n+1} = z_n^2 + c, \tag{1.3}$$

when $a_n \equiv 1, \ c_n \equiv 0$ and $c_{n+1} \equiv -c$, which was used to study its Julia set originally.

Setting $c = -0.5(1+i)$, the Julia set of system (1.3) has the form showed in Figure 1.

We can see that this form of the Julia set of system (1.3) is irregular, which implies that its stable domain is also irregular. What's more, we should note that system (1.3) is a special form of system (1.2).

For achieving certain requirements of stability for the systems, we need to obtain the regular domains of their Julia sets. As a result, we will focus on studying the regular formats of Julia sets for system (1.2) in this article.

2. The formats of Julia sets for system (1.2)

We will focus on the formats of Julia sets for system (1.2). Here we use the symbol $\mathcal{J}(1.2)$ to denote the Julia sets of system (1.2).