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# Homoclinic Cycle and Homoclinic Bifurcations of a Predator-prey Model with Impulsive State Feedback Control

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**Abstract** In this paper, the homoclinic bifurcation of a predator-prey system with impulsive state feedback control is investigated. By using the geometry theory of semi-continuous dynamic systems, the existences of order-1 homoclinic cycle and order-1 periodic solution are obtained. Then the stability of order-1 periodic solution is studied. At last, an example is presented to illustrate the main results.

**Keywords** Semi-continuous dynamic system, Successor function, Order-1 homoclinic cycle, Homoclinic bifurcation, Order-1 periodic solution.

**MSC(2010)** 34C23, 34C37, 34D15.

## 1. Introduction and model formulation

Since the mid 1920s, Vito Volterra and Alfred James Lotka proposed a groundbreaking model of the interaction between predators and prey [8, 17], researchers have conducted extensive research on predation, reciprocity, and competition mechanisms in recent years. A common research method is to study the evolutionary relationship between predators and prey by establishing suitable mathematical models. Then many mathematical models consisting of differential equations have been established and studied [3, 9, 15, 19, 20, 24, 26]. Some of them are represented by impulsive differential equations [11, 12, 21, 23]. Impulsive differential equations are a basic model for studying the process of a sudden change in the state of a system variable [1, 6, 27]. This sudden change is called a pulse. Systems with pulses that depend on the value of a variable in the systems are called the state-dependent impulsive system, which has become an important topic of impulsive differential equations and has been widely concerned by researchers [4, 5, 7, 13, 14, 16, 18, 22, 28–30].

Cui and Chen [2] proposed a mathematical model with functional response and undercrowding effect as follows,

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{a}{k}x(x-L)(k-x) - \dfrac{bxy}{1+hx}, \\ \dfrac{dy}{dt} = -cy + \dfrac{dxy}{1+hx}, \end{cases} \tag{1.1}$$

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where $x, y$ represent the population density of prey and predator, respectively. $a, L, k, b, h, c, d$ are positive constants and $L < k$.

Yuan [25] considered the crowding effect in predator and a simple functional response based on system (1.1) as follows,

$$\begin{cases} \dfrac{dx}{dt} = ax(x-L)(k-x) - bxy, \\ \dfrac{dy}{dt} = -cy + dxy - \alpha y^2, \end{cases} \tag{1.2}$$

where $x, y$ represent the population density of prey and predator. $b$ represents the predation rate, $c$ is the natural mortality rate of predators, $d$ is the rate at which predator takes prey and then converts it to its own growth. $\alpha$ is death rate due to crowding effect. We nondimensionalize the system (1.2) with the following scaling,

$$t = \frac{\tau}{ak^2}, x = ku, y = \frac{ak^2}{b}v, p = \frac{L}{k}, m = \frac{kd}{c}, n = \frac{a\alpha k^2}{bc}, r = \frac{c}{ak^2}.$$

For the sake of convenience, we still use $t$ to denote the change of time, then the system (1.2) will be transferred to

$$\begin{cases} \dfrac{du}{dt} = u(u-p)(1-u) - uv, \\ \dfrac{dv}{dt} = -rv(1 - mu + nv), \end{cases} \tag{1.3}$$

In this paper, we consider the pulse state feedback control system based on model (1.3) as follows,

$$\begin{cases} \left.\begin{aligned} &\frac{du}{dt} = u(u-p)(1-u) - uv, \\ &\frac{dv}{dt} = -rv(1 - mu + nv), \end{aligned}\right\} v \neq h, \\ \left.\begin{aligned} &\Delta u = -q_1 u, \\ &\Delta v = -q_2 v, \end{aligned}\right\} v = h. \end{cases} \tag{1.4}$$

It is obvious that $0 < p < 1$. $\Delta u = u(t^+) - u(t), \Delta v = v(t^+) - v(t)$. Considering the biological meaning, we will consider the solution of system (1.3) in region $R_+^2 = \{(x, y)|x \geq 0, y \geq 0\}$.

The organization of this paper is as follows. Some definitions and lemmas are presented in the section 2. We qualitatively analyze the system (1.3) in section 3. In section 4, we consider the existence and stability of order one periodic solution of system (1.4). In section 5, numerical simulations are carried out to illustrate the analytical results. We give a brief conclusion in section 6.

## 2. Preliminaries

In this section, we will introduce some notations, definitions and lemmas of the geometric theory of semi-continuous dynamic system, which will be useful for the following discussions. The following definitions and lemmas of semi-continuous dynamic system come from Chen et al. [1], Wei and Chen [22] and Pang and Chen [10].