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Abundant Exact Explicit Solutions to a Modified cKdV Equation*

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Abstract In this paper, we construct abundant exact explicit solutions to a modified cKdV equation by employing the three forms of (ω/g)-expansion method, i.e., (g'/g^2)-expansion method, (g'/g)-expansion method and (g')-expansion method. The solutions obtained are under different constraint conditions and are in the form of hyperbolic, trigonometric and rational functions, respectively, including kink (antikink) wave solutions, singular wave solutions and periodic singular wave solutions which have potential applications in physical science and engineering.

Keywords Modified cKdV equation, Exact explicit solutions, (ω/g)-expansion method, (g'/g^2)-expansion method, (g'/g)-expansion method, (g')-expansion method.

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1. Introduction

It is well known that the investigation of nonlinear wave equations and their solutions has been the field under discussion in different branches of engineering, physics and mathematics [1,2]. Many famous models, such as the Korteweg-de Vries (KdV) equation [3] and the Camassa-Holm (CH) eqation [4], have been proposed in different fields such as physics, chemistry, biology, mechanics, optics, etc. Among the study of these nonlinear models, their traveling wave solutions have gained considerate attention and a number of powerful methods have been developed to find their exact traveling wave solutions, such as the inverse scattering method [5], the Hirota's bilinear method [6], the Bäcklund transformation method [7], the bifurcation method of dynamical systems [8–18], the (g'/g)-expansion method [19] and so on.

In this paper, we aim to consider the following modified coupled Korteweg-de Vries (cKdV) equation [20],

$$\begin{cases} u_t = v_x - \frac{3}{2}uu_x + \alpha u_x, \\ v_t = \frac{1}{4}u_{xxx} - vu_x - \frac{1}{2}uv_x + \alpha v_x, \end{cases} \tag{1.1}$$

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where α is a constant.

When $\alpha = 0$, Eq.(1.1) is reduced to the cKdV equation, which is a general example of N-component systems, energy dependent schrödinger operators and bi-Hamiltonian structures for multi-component systems [20–23]. Many important equations, such as classical Boussinesq equation and the systems governing second harmonic generation, are connected to the cKdV equation through nonsingular transformations [20], which potentially enables solutions of cKdV equations to be interpreted in the context of these related equations. Therefore, because of its great importance, we will further study the exact explicit traveling wave solutions to Eq.(1.1). More preciously, we exploit the three forms of (ω/g)-expansion method [24], i.e., (g'/g^2)-expansion method, (g'/g)-expansion method and (g')-expansion method to obtain exact explicit expressions of traveling wave solutions to Eq.(1.1).

The rest of the paper is organized as follows. Section 2 is devoted to the description of the (ω/g)-expansion method. In Section 3, we apply the three forms of (ω/g)-expansion method to obtain exact explicit traveling wave solutions to Eq.(1.1). Finally, the paper ends with a brief conclusion.

2. Description of (ω/g)-expansion method

Suppose that a nonlinear equation, say in two independent variables x and t, is given by

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \cdots) = 0, \tag{2.1}$$

where $u = u(x,t)$ is an unknown function, P is a polynomial of u and its various partial derivatives. Now we briefly show the main steps of the (ω/g)-expansion method.

Step 1 Suppose that $u(x,t) = u(\xi)$ with $\xi = x - ct$, where c is a parameter to be determined later, then from Eq.(2.1), one obtains

$$P(u, u', -cu', u'', -cu'', c^2u'', \cdots) = 0. \tag{2.2}$$

Step 2 Suppose the solutions of Eq.(2.2) can be expressed by a polynomial of (ω/g) as follows

$$u(\xi) = \sum_{i=0}^{n} a_i \left(\frac{\omega}{g}\right)^i,\ a_n \neq 0, \tag{2.3}$$

and $\omega,\ g$ satisfy the relation

$$\left(\frac{\omega}{g}\right)' = b_0 + b_1\left(\frac{\omega}{g}\right) + b_2\left(\frac{\omega}{g}\right)^2, \tag{2.4}$$

where $b_0,\ b_1$ and b_2 are arbitrary constants.

Step 3 By substituting Eq.(2.3) into Eq.(2.2), making use of (2.4), and setting the coefficients of all powers of (ω/g) to be zeros, we will get a system of algebraic equations, from which c and $a_1, a_2, \cdots, a_n$ can be obtained explicitly.

Step 4 Substituting c and $a_1, a_2, \cdots, a_n$ obtained in Step 3 into Eq.(2.3), one get the possible solutions.