Journal of Nonlinear Modeling and Analysis
Volume 3, Number 2, June 2021, 193–207

http://jnma.ca; http://jnma.ijournal.cn
DOI:10.12150/jnma.2021.193

Bifurcations and New Traveling Wave Solutions for the Nonlinear Dispersion Drinfel'd-Sokolov $(D(m,n))$ System*

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Abstract In this paper, we employ the theory of the planar dynamical system to investigate the dynamical behavior and bifurcations of solutions of the traveling systems of the $D(m,n)$ equation. On the basis of the previous work of the reference [17], we obtain the solitary cusp waves solutions (peakons and valleyons), breaking wave solutions (compactons) and other periodic cusp wave solutions. Morever, we make a summary of exact traveling wave solutions to the $D(m,n)$ system including all the solutions which have been found from the references [4,14,17].

Keywords $D(m,n)$ system, Solitary wave solution, Periodic wave solution, Compacton, Peakon.

MSC(2010) 35B10, 35C07, 35C08.

1. Introduction

In this paper, we consider the following nonlinear dispersion Drinfel'd-Sokolov system ($D(m,n)$ system)

$$q_t + k(r^m)_x = 0, \tag{1.1a}$$
$$r_t + a(r^n)_{xxx} + bq_x r + cqr_x = 0, \tag{1.1b}$$

where m, n are positive integer, a, b, c, k are real valued constants. The physical application of this model can be seen in references [6,7] and the references therein.

For the coupled system, Biswas et al. [1] and Chen et al. [3] obtained the three kinds of solutions when (m,n)=(1,1) by using the method of exponential function and dynamical systems method respectively. In 2011, Ebadi et al. [5] obtained one-soliton solution of the $D(m,n)$ equation by the ansatz method. Xie et al. [14] have obtained the compact and solitary patterns solutions of the $D(m,n)$ equation by using the sine-cosine method and the sinh-cosh method. Deng et al. [4] have studied some particular travelling wave solutions of the $D(m,n)$ equation by using the Weierstrass elliptic function expansion method. Zhang et al. [17] have obtained

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*The authors were supported by National Natural Science Foundation of China (No. 11761075).

some travelling wave solutions by using the bifurcation theory of planar dynamical systems.

Though many authors have studied the $D(m,n)$ system, they only obtained relatively partial exact solutions. The paper will be further to study the issue on the basis of the previous work. We make some complement, expansion and summary to the solutions of the coupled system (1.1). Moreover, by using the bifurcation method of planar systems and simulation method of differential equations ([8–13, 15, 16]), we shall study the exact explicit bounded travelling wave solutions of (1.1) under different parameter condition.

2. Transformed equations

Using the traveling wave assumption that

$$q(x,t) = \phi(\xi), \tag{2.1a}$$

$$r(x,t) = \psi(\xi), \tag{2.1b}$$

where $\xi = x - \lambda t$ is the real parameter and λ is the wave speed, substituting (2.1) into Equations.(1.1), we can reduce Equations (1.1) to the following ODEs:

$$-\lambda\phi' + k(\psi^m)' = 0, \tag{2.2a}$$

$$-\lambda\psi' + a(\psi^n)''' + b\phi'\psi + c\phi\psi' = 0. \tag{2.2b}$$

Integrating (2.2a) once and letting integral constant to be zero, we have

$$\phi = \frac{k}{\lambda}\psi^m. \tag{2.3}$$

Inserting (2.3) to Equation (2.2b) yields

$$-\lambda\psi' + a(\psi^n)''' + \frac{k}{\lambda}(bm+c)\psi^m\psi' = 0. \tag{2.4}$$

Integrating (2.4) once, we obtain

$$\frac{\lambda}{a}\psi - n(n-1)(\psi^{n-2})\psi'^2 - n\psi^{n-1}\psi'' - \frac{k(bm+c)}{a\lambda(m+1)}\psi^{m+1} + A = 0, \tag{2.5}$$

which is equivalent to 2-dimensional Hamiltonian system

$$\frac{d\psi}{d\xi} = y, \quad \frac{dy}{d\xi} = \frac{-n(n-1)\psi^{n-2}y^2 + \alpha\psi - \frac{\beta(bm+c)}{m+1}\psi^{m+1} + A}{n\psi^{n-1}}, \tag{2.6}$$

where A is a integrating constant and $\alpha = \frac{\lambda}{a}$, $\beta = \frac{k}{a\lambda}$ with the first integral

$$H(\psi, y) = \frac{n}{2}\psi^{2(n-1)}y^2 - \frac{A}{n}\psi^n - \frac{\alpha}{n+1}\psi^{n+1} + \frac{\beta(bm+c)}{(m+1)(m+n+1)}\psi^{m+n+1} = h. \tag{2.7}$$