East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.030920.251220

Vol. **11**, No. 2, pp. 406-420
May 2021

Integro Cubic Splines on Non-Uniform Grids and Their Properties

T. Zhanlav[1] and R. Mijiddorj[2,*]

[1]*Institute of Mathematics and Digital Technology, Mongolian Academy of Sciences, Ulaanbaatar, Mongolia.*
[2]*Department of Informatics, Mongolian National University of Education, Ulaanbaatar, Mongolia.*

Received 3 September 2020; Accepted (in revised version) 25 December 2020.

Abstract. Integro cubic splines on a non-uniform grid using the integral values of an unknown function are constructed. We establish a consistency relation for integro cubic spline and derive a local integro cubic spline on non-uniform partitions. Approximation and convexity properties of the local integro cubic splines are also studied.

AMS subject classifications: 65D05, 65D07

Key words: Integro cubic spline, local construction, non-uniform grid, error analysis.

1. Introduction

Researchers from our university investigating the location of a robot equipped with a rotary encoder device, wanted us to determine the velocity $v(t)$ of the wheel of the rotary encoder, which registers the time series when it runs a constant distance or Area=□, cf. Fig. 1.

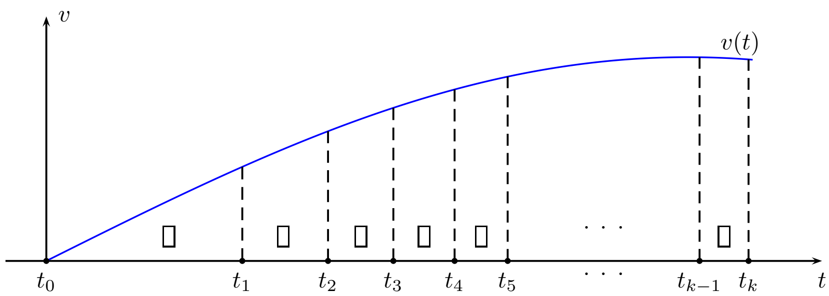

Figure 1: Time series registered with a rotary encoder.

*Corresponding author. *Email address:* `mijiddorj@msue.edu.mn` (R. Mijiddorj)

Of course, the problem of determining $v(t)$ concerns the histo-spline and the integro spline. There are many papers constructing integro splines [1–4, 10, 16, 17], but they are mainly focus on uniform partitions. Wu and Zhang [9] suggested an integro quadratic spline and Kirsiaed *et al.* [7] constructed a cubic spline histopolation on a non-uniform partition and studied its approximating properties. However, such splines do not solve the above problem, since the corresponding construction requires full information for a specified interval $[t_0, t_k]$. We have to provide real-time velocity $v(t)$ so that in our situation the local construction of the integro spline is more suitable. It is well-known [14] that the local construction of an integro spline has a lower computational cost than the constructions based on solving systems of linear equations.

This work is organised as follows. In Section 2, we consider an integro cubic spline on a non-uniform grid. Section 3 discusses the local construction of the integro cubic spline. In Section 4, we study the errors and convexity property of the spline proposed. Numerical examples presented in the last section illustrate the accuracy of the methods used.

2. Construction of Integro Cubic Splines

Let $\mathscr{T}_k := \{t_0 < t_1 < \ldots < t_k\}$ be a non-uniform partition of $[t_0, t_k]$ and $h_{i+1} = t_{i+1} - t_i$ are the step sizes. We have no information about the values of the function $v(t)$. However, it is known that for any subinterval $[t_i, t_{i+1}]$ the area □ under the graph of $v(t)$ is the same.

The problem of construction of integro cubic spline consists in finding an $S(t)$ such that the following conditions hold:

(i) On each subinterval $[t_i, t_{i+1}]$, $S(t)$ coincides with a polynomial of degree three.

(ii) $\displaystyle \frac{1}{h_i}\int_{t_{i-1}}^{t_i} S(t)dt = \frac{1}{h_i}\int_{t_{i-1}}^{t_i} v(t)dt = I_i, \quad i = 1, 2, \ldots, k.$

It follows from Fig. 1 and condition (ii) that □ $= h_i I_i$. We denote by $S_3(\mathscr{T}_k)$ the space of cubic splines over the partition $\mathscr{T}_k$, i.e.

$$S_3(\mathscr{T}_k) = \left\{p(x) | p(x) \in C^2[t_0, t_k]\right\},$$

where $p(x)$ is a polynomial of degree at most three on $\mathscr{T}_k$. According to [11], the elements $S \in S_3(\mathscr{T}_k)$ can be represented in one of the forms

$$S(t) = (1-\xi)^2(1+2\xi)S_{i-1} + \xi^2(3-2\xi)S_i + h_i\xi(1-\xi)\left\{(1-\xi)S'_{i-1} - \xi S'_i\right\}, \tag{2.1}$$

or

$$\begin{aligned} &S(t) = (1-\xi)S_{i-1} + \xi S_i - \frac{h_i^2}{6}\xi(1-\xi)\left[(2-\xi)S''_{i-1} + (1+\xi)S''_i\right], \\ &t \in [t_{i-1}, t_i], \quad \xi = \frac{t - t_{i-1}}{h_i}, \quad \xi \in [0, 1], \end{aligned} \tag{2.2}$$