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THE EFFECT OF REFUGE AND PROPORTIONAL HARVESTING FOR A PREDATOR-PREY SYSTEM WITH REACTION-DIFFUSION*†

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Abstract

A diffusive predator-prey system with Holling-Tanner functional response and no-flux boundary condition is considered in this work. By using upper and lower solutions combined with iteration method, sufficient condition which ensures the global asymptotical stability of the unique positive equilibrium of the system is obtained. It is shown that the prey refuge and the proportional harvesting can influence the global asymptotical stability of unique positive equilibrium of the system, furthermore, they can change the position of the unique interior equilibrium and make species coexist more easily.

Keywords reaction-diffusion system; iteration method; global asymptotical stability; prey refuge; proportional harvesting

2000 Mathematics Subject Classification 35K57

1 Introduction

To accurately describe the real ecological interactions between some species such as lynx and hare, mite and spider mite, sparrow and sparrow hawk, etc. described by Wollkind et al. [1] and Tanner [2], Robert May proposed a Holling-Tanner predator-prey model [3], in which the author incorporated Holling's rate [4,5]. In [6], Hsu and Huang studied the following predator-prey system

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = ru\Big(1-\dfrac{u}{K}\Big) - vp(u), \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = v\Big[s\Big(1-\dfrac{hv}{u}\Big)\Big], \\ u(0)>0, \quad u(0)>0, \end{cases} \tag{1.1}$$

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where u and v are the populations of the prey and the predator respectively. K is the carrying capacity of the prey and r is the intrinsic growth rate in the absence of predation. s is the intrinsic growth rate of the predator and $p(u)$ is the functional response. The carrying capacity of the predator is proportional to the population size of the prey. By using Dulacs criterion and constructing Liapunov functions, they established the global stability of the positive locally asymptotically stable equilibrium of system (1.1). For more biological background of system (1.1), one could refer to [6-8] and the references cited therein.

Taking into account the distribution of the prey and predators in spatial location within a fixed bounded domain $\Omega \subset R^N$ $(N \leq 3)$, Wonlyul Ko and Kimun Ryu [9] considered a Holling-Tanner predator-prey system with reaction-diffusion. In [10], Peng and Wang studied the following system

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1 \triangle u + u\Big(a - u - \dfrac{v}{m+u}\Big), & x \in \Omega,\ t > 0, \\ \dfrac{\partial v}{\partial t} = d_2 \triangle v + bv - \dfrac{v^2}{\gamma u}, & x \in \Omega,\ t > 0, \\ \dfrac{\partial u}{\partial \mathbf{n}} = \dfrac{\partial v}{\partial \mathbf{n}} = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x) > 0, \quad v(x,0) = v_0(x) \geq (\not\equiv) 0, & x \in \Omega, \end{cases} \tag{1.2}$$

where $u(x,t)$ and $v(x,t)$ are the species densities of the prey and predator respectively. The constants d_i $(i = 1, 2)$ are the diffusion coefficients of prey and predator respectively. $\mathbf{n}$ is the outward unit normal vector on the smooth boundary $\partial\Omega$. The initial datas $u_0(x)$ and $v_0(x)$ are continuous functions on $\overline{\Omega}$, the homogeneous Neumann boundary condition means that the system is self-contained and has no population flux across the boundary $\partial\Omega$. Obviously, as mentioned in [10], the above system has a unique coexisting positive equilibrium $(u, v) = (\overline{u}, \overline{v})$, where

$$\overline{u} = \frac{1}{2}\big\{a - m - b\gamma + \sqrt{(a - m - b\gamma)^2 + 4am}\big\}, \quad \overline{v} = b\gamma\overline{u}. \tag{1.3}$$

They studied the stability of the positive constant solution of system (1.2) and obtained sufficient conditions for the global stability of the positive equilibrium by constructing a suitable Lyapunov function. For the ecological sense of system (1.2) we can refer to [10] and the references cited therein.

Recently, Chen and Shi [11] reconsidered the above system (1.2), and proved that if

$$m > b\gamma \tag{1.4}$$

holds, then the unique constant equilibrium of system (1.2) is globally asymptotically