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TRAVELING WAVE SOLUTIONS FOR A PREDATOR-PREY MODEL WITH BEDDINGTON-DEANGELIS FUNCTIONAL RESPONSE*†

Zhuanling Gao, Xiangkui Zhao‡, Zhihong Zhao

(*School of Mathematics and Physics, University of Science & Technology Beijing, Beijing 100083, PR China*)

Abstract

In this paper, we study a class of predator-prey models with Beddington-DeAngelis functional response. And the predator equation has singularity in zero prey population, where a smoothing auxiliary function is introduced to overcome it. Our aim is to see if the predator and prey can eventually survive when an alien predator enters the habitat of an existing prey by employing traveling wave solutions, based on the upper and lower solutions and Schauder's fixed point theorem. In addition, the non-existence of traveling wave solutions is discussed by the comparison principle. At the same time, some simulations are carried out to further verify the results.

Keywords predator-prey model; Beddington-DeAngelis; traveling wave solution; existence

2000 Mathematics Subject Classification 35K10; 35K57

1 Introduction

In recent years, we have found that some species are endangered by their predators or other reasons. This will cause ecologically bankrupt. Therefore we pay more attention to it. The predator-prey model is an important tool to study the relationship of several species. And it is a topic attracting more and more attention from mathematicians and ecologists [3, 5–7, 12, 16]. So it has very broad application prospects. Whether the predator and prey can survive eventually is equivalent to the existence of traveling wave solution for specific model. So the traveling wave solution has studied by many scholars, see [1, 9, 10, 15, 19, 20] and their references.

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‡Corresponding author. E-mail: xiangkuizh@ustb.edu.cn

In 2016, Chen, Yao, and Guo [4] studied the diffusion predator-prey model of Lotka-Volterra type functional response. The model is as follows:

$$\begin{cases} u_t = u_{xx} + ru(1-u) - rkuv, \\ v_t = dv_{xx} + sv\Big(1 - \dfrac{v}{u}\Big), \end{cases} \tag{1.1}$$

where u, v represent the population densities of the prey and predator species at position x and time t, respectively, d, r, s, k are constant numbers, d is the diffusion coefficient, and r, s are the intrinsic growth rates of species u, v, respectively. The functional response of the predator to the prey is given by the Lotka-Volterra type rku.

Then, Zhao [20] studied the model as $k = 1$, namely

$$\begin{cases} u_t = u_{xx} + ru(1-u) - ruv, \\ v_t = dv_{xx} + sv\Big(1 - \dfrac{v}{u}\Big). \end{cases} \tag{1.2}$$

In 2017, Ai, Du, Peng [1] studied the traveling wave solution of the Holling-Tanner predator-prey model:

$$\begin{cases} u_t = d_1 u_{xx} + u(1-u) - \dfrac{\alpha u^m}{1+\beta u^m} v, \\ v_t = d_2 v_{xx} + rv\Big(1 - \dfrac{v}{u}\Big), \end{cases} \tag{1.3}$$

where u, v represent the population densities of the prey and the predator at position x and time t, respectively, the parameters α, m and r are positive and β is non-negative. Here, the predation rate in the prey equation is controlled by a so-called Holling type functional response. The predator equation is also singular at zero prey population. When $m = 1, \beta = 0$, model (1.3) becomes model (1.2).

The functional response function of (1.3) is a class of Holling-II type functional response function only depending on the prey. However, in reality, it is not independent of predator either. In fact, the B-D functional response function $\Phi(u, v) = \frac{cu}{m_0 + m_1 u + m_2 v}$ maintains all the advantages of the ratio-dependent response function and avoids the controversy caused by the low-density problem, so it can better reflect the real relationship of the two species [8, 11, 17, 18]. When $m_1 = 0$ and $m_2 \neq 0$, the Beddington-DeAngelis type functional response is also a class of Holling-II type [13, 14].

We will consider the following model:

$$\begin{cases} U_t = U_{xx} + U(1-U) - \dfrac{\alpha U}{1+\beta_1 U + \beta_2 V} V, \\ V_t = dV_{xx} + rV\Big(1 - \dfrac{V}{U}\Big), \end{cases} \tag{1.4}$$