Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **13**, No. 3, pp. 724-734

DOI: 10.4208/aamm.OA-2019-0307
June 2021

Nonlinear Hydroelastic Waves Traveling in a Plate in Terms of Plotnikov–Toland's Model

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Received 22 October 2019; Accepted (in revised version) 2 September 2020

Abstract. Analytical study on nonlinear hydroelastic waves beneath a very large floating structure or a thin ice sheet floating on deep water is presented. Adopting the special Cosserat theory of hyperelastic shells satisfying Kirchhoff's hypothesis to describe the floating sheet, we use the potential flow theory with the dynamic boundary condition expressing a balance among the hydrodynamic, surface tension, inertial, and elastic forces. For the case of incident progressive waves, the influences of different physical parameters on the hydroelastic waves are discussed with the aid of the homotopy analysis method. We compare the hydroelastic wave deflections based on nonlinear Plotnikov and Toland's model with those obtained by the corresponding linear Euler–Bernoulli model. It is found that the behaviors of both models are almost the same for small amplitudes, while the nonlinear plate deflections increase greatly at large amplitudes. Further, the graphical comparisons are presented to show the behavior of the angular frequency versus wave amplitudes.

AMS subject classifications: 74J30, 76B07

Key words: Nonlinear hydroelastic waves, incident progressive waves, special Cosserat theory of hyperelastic shells, homotopy analysis method.

1 Introduction

Hydroelastic wave traveling beneath a very large floating structure (VLFS) or an ice sheet, which is usually studied by an elastic plate or beam model, has recently become a hot issue in the field of polar engineering and ocean engineering. A comprehensive summary

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on hydroelastic wave–plate interaction has been systematically described by some review articles such as [1–3].

A technical challenge in the hydroelastic problem is to model the VLFS or the ice sheet deflections in response to water wave excitations. To simplify the calculation, most studies use the linear Kirchoff–Love plate or Euler–Bernoulli beam theory to describe how the floating sheet bends in potential flow [2] for small-amplitude waves or small-amplitude plate deflections. For example, by modeling the ice floe as a linear Kirchoff–Love elastic plate, Meylan [4] found that ice floe stiffness is the most important factor in determining ice floe motion, scattering, and force. Wang and Lu [5] analytically investigated the nonlinear hydroelastic waves traveling in a floating elastic plate with the aid of the homotopy analysis method (HAM). Recently, Wang and Lu [6] extended their study [5] to a two-layer fluid of finite depth and investigated the effect of the fluid stratification on the nonlinear hydroelastic waves traveling in an infinite elastic plate. Unsteady hydroelastic waves caused by the interaction of fixed concentrated line loads and the underlying current were considered by Lu and Yeung [7]. Părău and Dias [8] studied the large deformations of a floating ice sheet induced by a moving load in which the sheet is modeled as a nonlinear Kirchoff–Love plate. The speed of gravity waves on shallow water and the minimum phase speed were investigated in details. For some generality, Hegarty and Squire [9] represented the plate mathematically using three different nonlinear mechanical models which are some extensions to the standard Kirchoff–Love plate formulation and incorporate different geometrically nonlinear terms.

Considering the floating elastic plate may have greater bending in the real deep-water ocean, some researches have adopted nonlinear models based on the special Cosserat theory of hyperelastic shells satisfying Kirchhoff's hypothesis. Plotnikov and Toland [10] applied the special Cosserat theory of hyperelastic shell model to study the interaction between a thin elastic sheet and an infinite ocean beneath it. Guyenne and Părău [11] further extended Plotnikov and Toland's model to two-dimensional nonlinear flexural–gravity waves propagating at the surface of an ideal fluid of finite depth. It is found that small- to large-amplitude solitary waves of depression are stable while its elevations seem to be unstable in all cases. Lately, with the help of Plotnikov and Toland's model, Bhatti and Lu [12] studied the head-on collision between two hydroelastic solitary waves propagating beneath a thin ice sheet by using a singular perturbation method. It is observed that the wave profile is symmetric before collision, and it becomes unsymmetric and titled backward in the direction of wave propagation after collision.

In this paper, we extend the study on hydroelastic waves by use of Plotnikov and Toland's nonlinear model [10]. Hydroelastic wave deflections for different physical parameters are analytically discussed with the aid of the HAM. We compare the influence of Plotnikov and Toland's model with that of the corresponding linear one for several different wave amplitudes. Finally, we figure out the variation of angular frequency versus wave amplitudes.