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CAN A CUBIC SPLINE CURVE BE G^3*

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Abstract

This paper proposes a method to construct an G^3 cubic spline curve from any given open control polygon. For any two inner Bézier points on each edge of a control polygon, we can define each Bézier junction point such that the spline curve is G^2-continuous. Then by suitably choosing the inner Bézier points, we can construct a global G^3 spline curve. The curvature combs and curvature plots show the advantage of the G^3 cubic spline curve in contrast with the traditional C^2 cubic spline curve.

Mathematics subject classification: 65D07.
Key words: Cubic Spline, Geometric Continuity, G^3 Continuity.

1. Introduction

Curve modeling has a long history in computer graphics, which is widely used in drawing, sketching, data fitting, interpolation, as well as animation. The basic goal of curve modeling is to provide the algorithm to edit the shape of the curve with some certain geometric properties. In industrial or conceptual design, one important question is how to construct a fair freeform curve. Most CAD systems rely on some sort of curvature information to define a fair curve and the prevailing tools are curvature combs and curvature plots [30,31]. The most used representations in the industry design are cubic B-splines. However, the curvature plot or curvature comb has some non-smooth junctions because the spline curve is at most C^2-continuous, as shown in Fig. 1.1. This leads a very nature question: can we increase the continuity of a cubic spline curve?

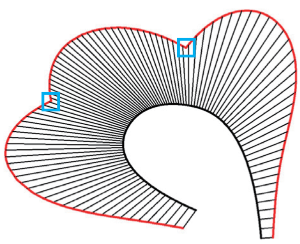

Fig. 1.1. The curvature comb of a cubic B-spline curve has some non-smooth junctions.

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This paper gives a positive answer for this problem and proposes a method to construct a G^3 cubic spline curve for any given open control polygon. First, for the given control polygon, we can select any two inner Bézier points on each edge and define the junction points to construct a global G^2 spline curve. And then, we can construct a global G^3 spline curve by solving the inner Bézier points. We prove that the solution always exists under a restriction on the control polygon. Same as the B-spline curve, the curve satisfies the convex hull property and can be modified by editing the control polygon. The curvature combs and curvature plots show the advantage of the G^3 cubic spline curve in contrast with the traditional C^2 cubic spline curve.

1.1. Related work

Among all the representations, Non-Uniform Rational B-Spline (NURBS) is the industry standard for the representation, design, and data exchange of geometric information processed by computers [2]. Many different models of splines have been introduced for various purpose. B-spline is introduced by Schoenberg [23] using divided difference which is not suitable for computing [3]. And then, C. de Boor [5] and M. Cox [6] discover the recurrence relations independently. The recurrence relation is used by Gordon and Riesenfeld [7] for efficient computing. The B-Spline can also be regarded as the basis functions of a linear spline space [4]. Beta-spline is introduced by Barsky [8] by replacing the derivatives with the tangent vector and curvature vector [9], which preserves the geometric smoothness of the curve and gives greater flexibility to control the shape [10, 11]. The definition of geometric continuity has been used by many other authors, such as Manning [12], Nielson [13], Barsky and DeRose [14] and Böhm [15]. Pythagorean hodograph (PH) spline curves are defined in [16], which provide rational offset curves and polynomial arc-length functions [17,18]. X-spline model is proposed in [19] to make user manipulations more intuitively. The other geometrical continuous spline curve constructions have been widely developed as well. Schaback [34] constructs a piecewise quadratic G^2 Bézier interpolatory curve by satisfying certain generalized convexity conditions. Yan et al. [33] construct almost everywhere curvature continuous piecewise quadratic curves, called κ–curves. Miura et al. [36] design log-aesthetic spline curves with G^2 continuity by solving the G^2 Hermite interpolation problem. Farin proposes the constructing of G^2 cubic spline with the given control polygon by using the Euclidean distances in [21,22]. The curvature continuous PH spline curves have been constructed from any control polygon in [32]. However, all of these constructions only involve G^2 continuity at most.

1.2. Organization

The rest of the paper is organized as follows. In Section 2, we recall some basic properties of B-spline and present the derivatives of a B-spline curve respecting the arc length. In Section 3, we derive the G^3 continuous condition and provide the algorithm to construct the G^3 cubic spline curves. We also prove that the solution always exists under a restriction on the control polygon. The examples of the present construction and traditional B-spline are shown in Section 4. Finally, Section 5 presents the conclusion and future work.